Self-dual [40, 20, 8] codes with covering radius 7

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s-extremal code of length 40, $d=6$, $s=12$

(unique)

is a neighbor of

Harada - Ozeki

extremal doubly-even self-dual (Type II) code of length 40, $d=8$

with covering radius 7

C : binary linear code $\subset \mathbb{F}_2^n$

Covering radius of C

$= \max\{\min(C+x) \mid x \in \mathbb{F}_2^n\}$

Then

sphere covering bound

$\leq$ covering radius $\leq$ Delsarte bound

For $C : [40, 20, 8]$ Type II (doubly-even self-dual)

$6 \leq$ covering radius ≤ 8

usually 8

Harada-Ozeki (2000) found 7

Harada-M-Tanabe found another 7

not known to exist 6

A description of Harada-Ozeki's code D

$G = [24, 12, 8]$ Golay code

$\Rightarrow$ G_{20} = 4-ply shortened $[20, 8, 8]$ code

$\Rightarrow$ $S = \{ (u \mid u+v) \mid u \in G_{20}^{\perp},\ v \in G_{20} \}$

: $[40, 20, 8]$ Type I code

$\Rightarrow$ D = a Type II neighbor of S

$[40, 20, 8]$ Type II code

D has covering radius 7

(First $[40, 20, 8]$ Type II code with covering radius 7 ever found)

A coset of D

Harada-Ozeki observed:

$\exists$ coset $D+v$ with $\min(D+v)=6$
$(D+v)_8 = \phi$

There are only 6 such cosets (out of 2^{20}). Each such coset supports 1-designs.

Theorem Let D be a $[24m+16, 12m+8, 4m+4]$ Type II code. Suppose $\exists$ coset $D+v$ such that $\min(D+v)=4m+2$, $(D+v)_{4m+4}=\phi$

Then $D+v$ supports 1-designs

D = Harada-Ozeki $[40, 20, 8]$ code

$\min(D+v) = 6, \quad (D+v)_8 = \phi$

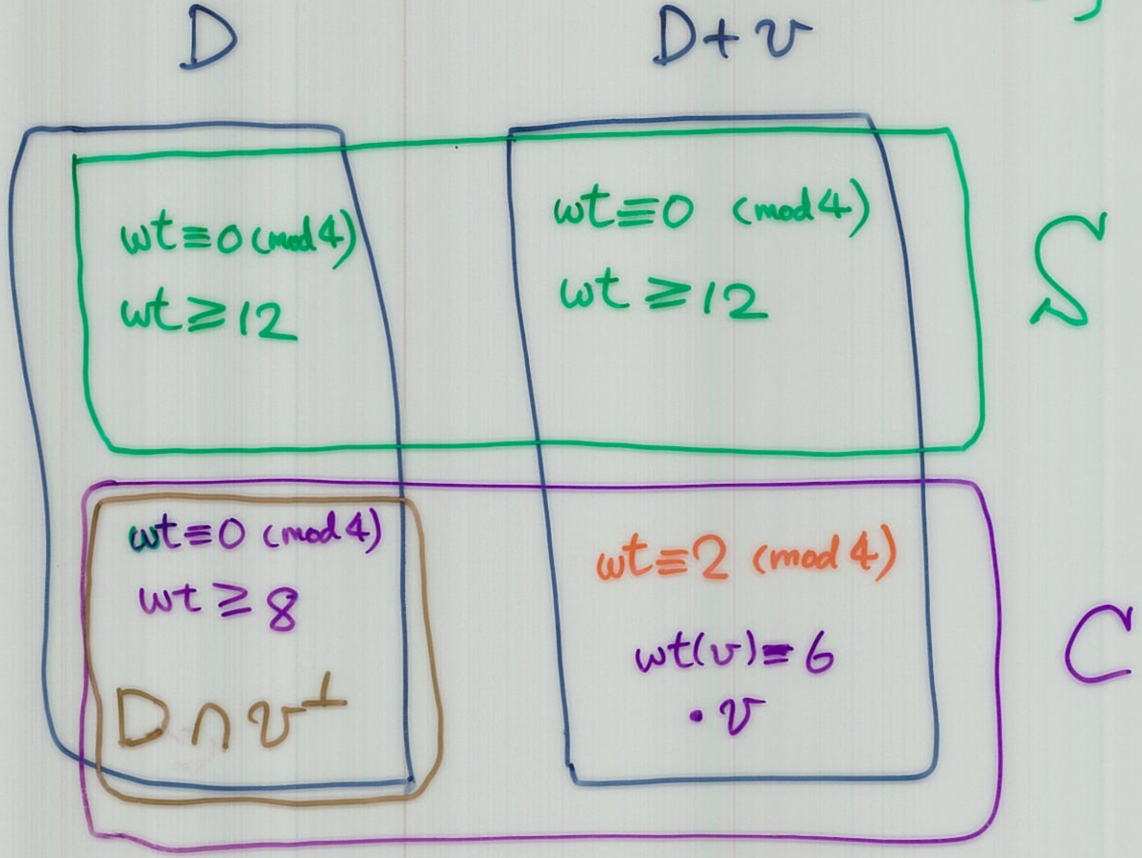
C = the neighbor of D w.r.t. v

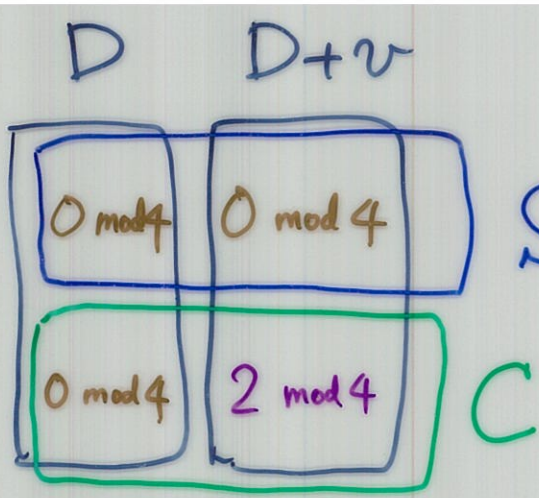
$\Rightarrow C$ = s-extremal Type I code

$d = 6, \quad s = 12 \qquad 2d + s = 24$

$= 4 + \frac{n}{2}$

[unique by Bachoc-Gaborit]



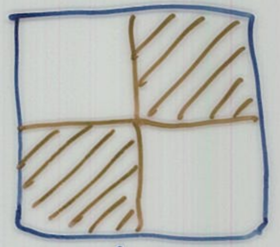


S = shadow of C

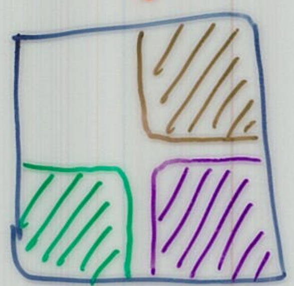
Harada-M-Tanabe

1-design

Assmus-Mattson

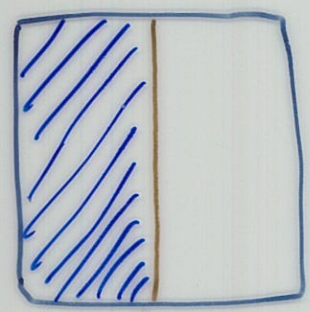


1-design

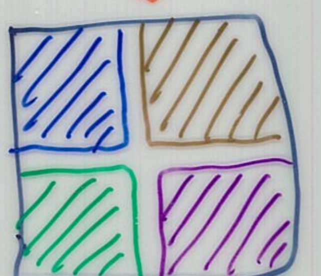


1-design

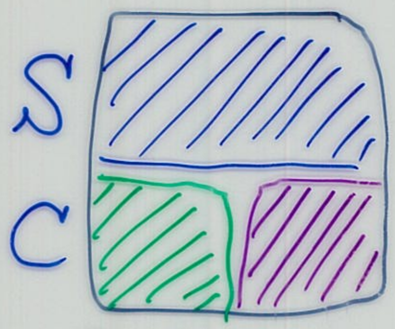
Assmus-Mattson



1-design



1-design



Bachoc-Gaborit