A

finite field ex. $\{0,1\}$

+	0	1
0	0	1
1	1	0

×	0	1
0	0	0
1	0	1

F additive gp
$F \setminus \{0\}$ mult. gp. } distributive.

F^n = vector space $/F$. $F = \mathbb{F}_2$ F^3 consists of 8 vectors.

~~000~~
001
⋮

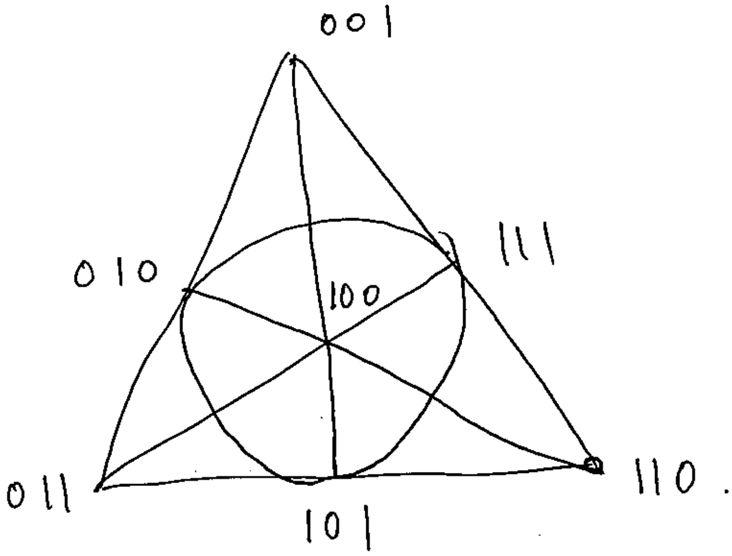
~~000~~
001
010
011
} closed under +. subspace, dim = 2.

subsp of dim 1 dim 2

pts line

001 — 001, 010, 011
010 — 001, 100, 101
011
100 ⋮
101
110
111

7



△ ∗ ○
7 subsp of dim 2

incidence graph
collinearity graph.

line graph $= K_7$.

1.

$F = GF(q)$

$V =$ vector space over F, $\dim V = n+1$

$PG(n-1, q) = \{ [v] \mid v \in V,\ v \neq 0 \}$

$[v] = \{ \lambda v \mid \lambda \in F,\ \lambda \neq 0 \}$ for $0 \neq v \in V$.

For $W \subset V$,

$[W] = \{ [v] \mid v \in W \}$ so $[V] = PG(n-1, q)$

$\begin{bmatrix} V \\ k \end{bmatrix} = \{ W \mid W \subset V,\ \dim W = k \}$

2

Example $n = 3$. $q = 2$.

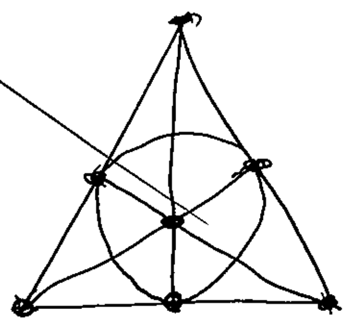
$V = F^3$

$PG(2,2) = \{ [\quad] \cdots [\quad] \}$

$W = \langle (0\,0\,1), (0\,1\,0) \rangle$

$= \{ (000), (001), (010), (011) \}$

$[W] = \{ [0\,0\,1], [0\,1\,0], [0\,1\,1] \}$



Grassman graph $J_q(n, e)$

$\begin{bmatrix} V \\ e \end{bmatrix}$ = vertices

$W_1, W_2 \in \begin{bmatrix} V \\ e \end{bmatrix}$ $\quad W_1 \sim W_2 \Leftrightarrow \quad \dim W_1 \cap W_2 = e-1$.

$J_q(n, e) \cong J_q(n, n-e)$

Design

points $= PG(n-1, q) = [V]$.

blocks $= \{ [W] \mid W \in \begin{bmatrix} V \\ e \end{bmatrix} \}$

$[0\ 0\ 1]$

$[0\ 1\ 0]$

$\{ \cancel{[001]}\ \cancel{[010]}\ [011] \}$

$1 \le e < n$.

$e = 1$.

$\Downarrow$

pts = blocks

$\Downarrow$

null graph

Assume

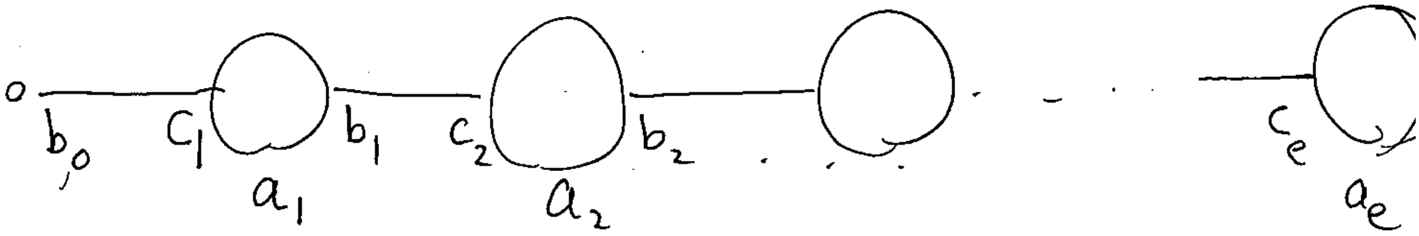
$2 \le e \le n-2$

~~blocks = vertices of graph.~~

intersecting pair of blocks = edges

Characterization

$J_q(n,e)$ is a DRG with int array



$$b_i = q^{2i+1}\begin{bmatrix} e-i\\ 1\end{bmatrix}\begin{bmatrix} n-e-i\\ 1\end{bmatrix}$$

$$c_i = \begin{bmatrix} i\\ 1\end{bmatrix}^2$$

$$a_i = b_0 - b_i - c_i$$

Question $J_q(n,e)$ characterized by its int array?

~~Koolen - van Dam, 2005~~

$n \neq 2e+2, 2e+1, 2e$

$\Rightarrow$ yes Metsch (1995)

$n = 2e+1$. van Dam-Koolen twisted

Jungnickel-Tonchev (2009)

design

points $[V]$ "distort"

~~blocks~~ $\longleftrightarrow \begin{bmatrix} V \\ e \end{bmatrix}$ "twist"

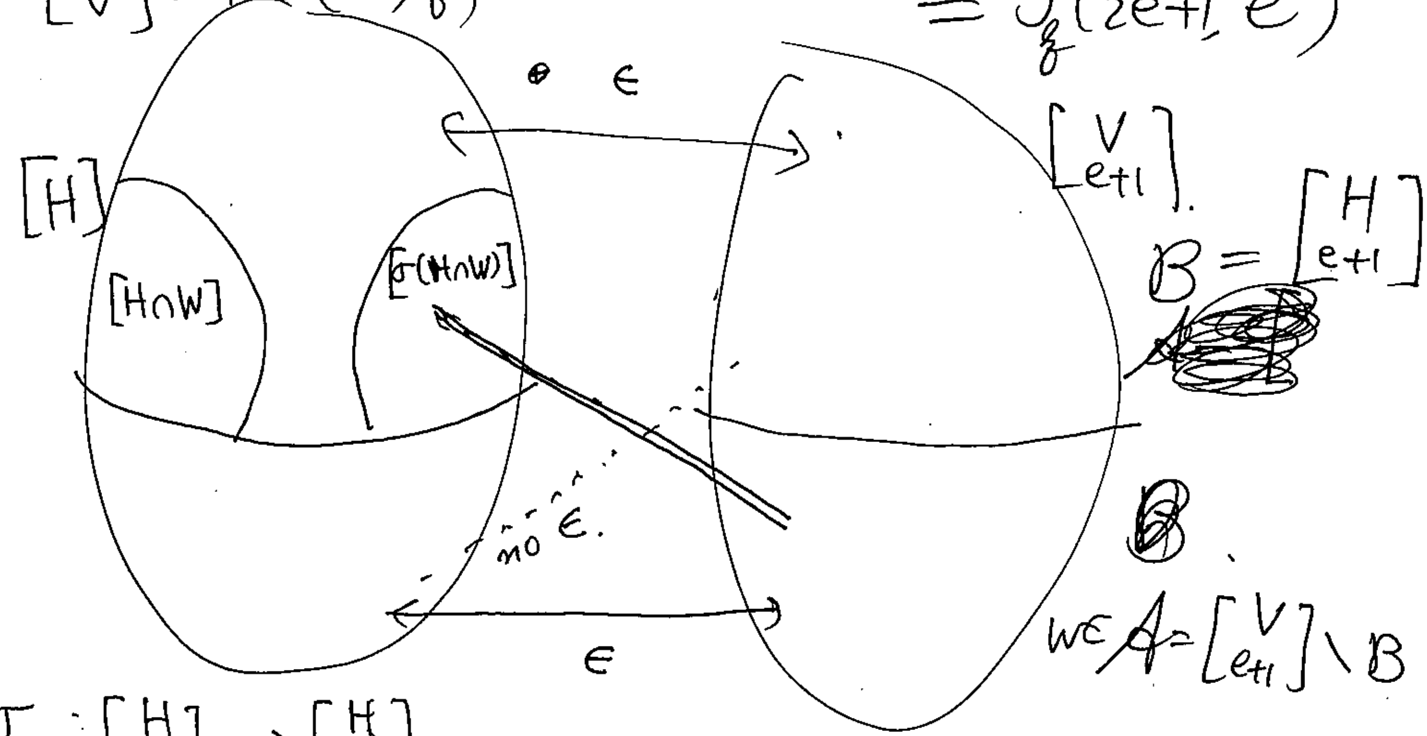
M.-Tonchev

the block graph of J-T design is the twisted G. graph

$n = \dim V = 2e+1$. $\quad H \in \begin{bmatrix} V \\ 2e \end{bmatrix}$.

consider $\begin{bmatrix} V \\ e+1 \end{bmatrix}$. $\quad J_q(2e+1, e+1)$

$[V] = PG(n-1, q)$ $\quad \cong J_q(2e+1, e)$



$\sigma : \begin{bmatrix} H \\ e \end{bmatrix} \longrightarrow \begin{bmatrix} H \\ e \end{bmatrix}$