On the Enumeration of Binary Self-Dual Codes

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Abstract

We give a classification of singly-even self-dual binary codes of length 32, by enumerating all neighbours of the known 85 doubly-even self-dual binary codes of length 32. There are $3,210$ singly-even self-dual binary codes of length 32 up to equivalence. This agrees in number with the enumeration by Bilous and van Rees, who enumerated these codes by a different method.

1 Preliminaries

We denote by $\mathbf{F}_2$ a finite field of two elements. A binary linear code of length n is a linear subspace of $\mathbf{F}_2^n$. Since all codes considered in this note are binary linear, we omit the adjectives "binary" and "linear." Two codes are said to be equivalent if one is obtained from the other by a permutation of coordinates. The set of permutations which give equivalences of a code C to itself forms a group called the automorphism group, and is denoted by $\mathrm{Aut}(C)$. We define the standard inner product on $\mathbf{F}_2^n$ by

$$u \cdot v = \sum_{j=1}^{n} u_j v_j, \quad (u, v \in \mathbf{F}_2^n).$$

The dual $C^\perp$ of a code C is defined by

$$C^\perp = \{u \in \mathbf{F}_2^n \mid u \cdot v = 0 \text{ for all } v \in C\},$$

and C is said to be self-dual if $C = C^\perp$. The weight of a vector $u \in \mathbf{F}_2^n$ is defined by

$$\mathrm{wt}(u) = \#\{j \mid 1 \le j \le n,\ u_j \ne 0\}.$$

and the minimum weight of a code C is defined by

$$\min(C) = \min\{\mathrm{wt}(u) \mid u \in C,\ u \neq 0\}.$$

If C is a self-dual code, then clearly $\mathrm{wt}(u) \equiv 0 \pmod 2$ holds for all $u \in C$. A code C is called doubly-even if for all $u \in C$, $\mathrm{wt}(u) \equiv 0 \pmod 4$ holds. A doubly-even self-dual code is also called a Type II code. A self-dual code which is not doubly-even is called singly-even or Type I. A Type II code of length n exists if and only if $n \equiv 0 \pmod 8$.

Now we turn to the enumeration of self-dual codes. All Type II codes of length up to 32 have been classified, and all Type I codes of length less than 32 have been classified, by Conway, Pless and Sloane [4]. The main result of this note is the following theorem, which in particular fills the last entry in Table 1.

Theorem 1. *There are* $731, 2402, 74, 3$ *Type I codes of length* 32 *with minimum weight* $2, 4, 6, 8$*, respectively, up to equivalence.*

Table 1: The Number of Self-Dual Codes

n	2	4	6	8	10	12	1	16	18	20	22	24	26	28	30	32
II				1				2				9				85
I	1	1	1	1	2	3	4	5	9	16	25	46	103	261	731	3210

The first row indicates the lengths, the second row gives the number of doubly-even self-dual (Type II) codes, and the third row gives the number of singly-even self-dual (Type I) codes, both up to equivalence. Among the Type I codes of length 32, only those codes with minimum weight 2 or 8 were classified before. Type I codes of length 32 with minimum weight 2 are the direct sums of self-dual codes of length 30 and the unique self-dual code of length 2, hence there are 731 such codes. It is known that there are three Type I codes of length 32 with minimum weight 8 (see [5]).

After our enumeration was completed, the author became aware of the work of Bilous and van Rees [1] who also obtained Theorem 1 by a method different from the one described in this note. The purpose of the present note now, is to demonstrate the efficiency of our approach. Also, it should be mentioned that our method of enumerating neighbours (see the next section for a definition) has been applied successfully in finding new quasi-symmetric 2-$(49, 9, 6)$ designs (see [6]).

The total number of Type II codes of length n is known to be (see [7]):

$$\prod_{j=0}^{n/2-2} (2^j + 1).$$

A similar formula for the total number of self-dual codes of length n is known:

$$\prod_{j=1}^{n/2-1} (2^j + 1).$$

These formulas can be used to check that a classification of Type I codes is complete for small lengths. In fact, suppose that we have a collection $\mathcal{C}$ of Type I codes of length n in which all members are pairwise inequivalent. If $\mathcal{C}$ satisfies

$$\sum_{C\in\mathcal{C}} \frac{n!}{\#\operatorname{Aut}(C)} = \prod_{j=1}^{n/2-1} (2^j + 1) - \prod_{j=0}^{n/2-2} (2^j + 1). \tag{1}$$

then we can be sure that no other Type I codes of length n exist, in other words, $\mathcal{C}$ is a complete set of representatives of equivalence classes of Type I codes of length n.

2 Neighbours

Self-dual codes C_1, C_2 of length n are called neighbours to each other if

$$\dim C_1 \cap C_2 = \frac{n}{2} - 1$$

Note $\dim C_1 = \dim C_2 = \frac{n}{2}$ since C_1 and C_2 are self-dual. The vector in $\mathbf{F}_2^n$ whose entries are all 1 is called the all one vector and is denoted by $\mathbf{1}$. Every self-dual code contains $\mathbf{1}$. Thus, a self-dual code C of length n has $2^{n/2-1} - 1$ hyperplanes (subcodes of codimension 1 in C) containing $\mathbf{1}$. For each such hyperplane H, $\dim H^{\perp}/H = 2$ holds. So there are three self-dual codes (including C) lying between H and $H^{\perp}$. Indeed, the following holds (see [3]).

Lemma 1. *Let n be a positive integer divisible by 8. Let D_0 be a doubly-even code of length n containing the all one vector, and suppose $\dim D_0 = n/2-1$. Then there are exactly three self-dual codes containing D_0, only one of which is Type I.*

Therefore, a Type II code C of length n has $2^{n/2-1}-1$ Type I neighbours and $2^{n/2-1}-1$ Type II neighbours. All Type I codes arise in this way from a Type II code and its hyperplane. Indeed, if C_1 is a Type I code, then there exists a unique doubly-even hyperplane H of C_1. The two neighbours C', C'' of C_1 such that

$$C_1 \cap C' = C_1 \cap C'' = H$$

are both Type II.

Thus we can use the classification of Type II codes of length 32 in order to classify Type I codes of length 32. More precisely, we need to classify the pairs (D, D_0), where D is a Type II code of length 32, D_0 is a hyperplane of D. Note that D can be chosen to be one of the 85 representatives of equivalence classes of Type II codes of length 32. Then D_0 can be chosen to be one of the representatives of orbits under $\mathrm{Aut}(D)$. Still, equivalent codes may arise from more than one pairs (D, D_0), hence equivalence testing must be performed in order to classify Type I codes. We will describe how to minimize the costly computation of equivalence testing in the next section. We will see that the number of equivalence classes of Type I codes can be found without performing any equivalence testing.

Let n be a positive integer divisible by 8, and let $\mathcal{C}$ denote the set of Type I codes of length n. Let $\mathcal{D}$ denote the set of pairs (D, D_0), where D is a Type II code of length n, D_0 a subcode of D of codimension 1 containing the all one vector. For each $(D, D_0) \in \mathcal{D}$, Lemma 1 implies that there exists a unique Type I code containing D_0. This fact enables us to define a mapping $\phi : \mathcal{D} \to \mathcal{C}$, where $C = \phi(D, D_0)$ is defined to be the unique Type I code containing D_0. Every Type I code has a uniquely determined doubly-even subcode of codimension 1. Hence Lemma 1 implies that ϕ is surjective and $|\phi^{-1}(C)| = 2$ for all $C \in \mathcal{C}$. Indeed, if D_0 is the doubly-even subcode of codimension 1 in C, then

$$\phi^{-1}(C) = \{(D, D_0), (D', D_0)\},$$

where D, D' are the two Type II codes containing D_0.

We now define three subsets of $\mathcal{D}$ as follows.

$$\begin{aligned}
\mathcal{D}_1 &= \{(D, D_0) \in \mathcal{D} \mid \mathrm{Aut}\, D_0 \not\subset \mathrm{Aut}\, D\}, \\
\mathcal{D}_2 &= \{(D, D_0) \in \mathcal{D} \mid D \not\cong D'\}, \\
\mathcal{D}_3 &= \{(D, D_0) \in \mathcal{D} \mid \mathrm{Aut}\, D_0 \subset \mathrm{Aut}\, D \text{ and } D \cong D'\},
\end{aligned}$$

where, in the definition of $\mathcal{D}_2$ and $\mathcal{D}_3$, D' denotes the unique Type II code containing D_0, different from D. Also, the symbol $\cong$ means the equivalence of codes.

Lemma 2. *Suppose* $(D, D_0) \in \mathcal{D}$*, and let* D' *denote the unique Type II code containing* D_0*, different from* D*. If* $(D, D_0) \in \mathcal{D}_1$*, then* $D \cong D'$*.*

Proof. Since $\operatorname{Aut} D_0 \not\subset \operatorname{Aut} D$, there exists an element $\sigma \in \operatorname{Aut} D_0$ such that $D^\sigma \neq D$. Then we must have $D^\sigma = D'$, and in particular, $D \cong D'$. □

Lemma 2 implies that $\mathcal{D}$ is the disjoint union $\mathcal{D}_1 \cup \mathcal{D}_2 \cup \mathcal{D}_3$. In fact one can prove the following.

Lemma 3. *We have*

$$\mathcal{C} = \phi(\mathcal{D}_1) \cup \phi(\mathcal{D}_2) \cup \phi(\mathcal{D}_3) \quad \textit{(disjoint).}$$

Proof. Let $C \in \mathcal{C}$, and suppose $\phi^{-1}(C) = \{(D, D_0), (D', D_0)\}$. If $(D, D_0) \in \mathcal{D}_1$, then there exists an element $\sigma \in \operatorname{Aut} D_0$ such that $D^\sigma \neq D$. This forces $D^\sigma = D'$, hence $\sigma \notin \operatorname{Aut} D'$. Therefore, $(D', D_0) \in \mathcal{D}_1$. It is clear that $(D, D_0) \in \mathcal{D}_2$ implies $(D', D_0) \in \mathcal{D}_2$. Thus $(D, D_0) \in \mathcal{D}_3$ forces $(D', D_0) \in \mathcal{D}_3$. Therefore, $\phi^{-1}(C)$ is contained in one of $\mathcal{D}_1, \mathcal{D}_2$ or $\mathcal{D}_3$. □

Let $\bar{\mathcal{C}}$ be the set of equivalence classes of Type I codes of length n. In other words,

$$\mathcal{C} = \{[C] \mid C \in \mathcal{C}\},$$

where

$$[C] = \{C^\sigma \mid \sigma \in S_n\}$$

is the S_n-orbit containing C. Then $\bar{\mathcal{C}}$ is a partition of $\mathcal{C}$. Likewise, S_n acts on $\mathcal{D}$, and $\mathcal{D}$ is partitioned into S_n-orbits. We denote by $[D, D_0]$ the S_n-orbit containing $(D, D_0) \in \mathcal{D}$, and set

$$\bar{\mathcal{D}} = \{[D, D_0] \mid (D, D_0) \in \mathcal{D}\}.$$

Then ϕ induces the mapping $\bar{\phi} : \bar{\mathcal{D}} \to \bar{\mathcal{C}}$ defined by $\bar{\phi}([D, D_0]) = [\phi(D, D_0)]$.

It is clear that each of the subsets $\mathcal{D}_1, \mathcal{D}_2$ and $\mathcal{D}_3$ is invariant under the action of S_n. Thus

$$\bar{\mathcal{D}} = \bar{\mathcal{D}}_1 \cup \bar{\mathcal{D}}_2 \cup \bar{\mathcal{D}}_3,$$

where

$$\bar{\mathcal{D}}_i = \{[D, D_0] \mid (D, D_0) \in \mathcal{D}_i\} \quad (i = 1, 2, 3).$$

The following is an immediate consequence of Lemma 3.

Lemma 4. *We have*

$$\bar{\mathcal{C}} = \bar{\phi}(\bar{\mathcal{D}}_1) \cup \bar{\phi}(\bar{\mathcal{D}}_2) \cup \bar{\phi}(\bar{\mathcal{D}}_3) \quad \textit{(disjoint).}$$

Moreover, we have the following.

Lemma 5. *For any $C \in \mathcal{C}$,*

$$|\bar{\phi}^{-1}([C])| = \begin{cases} 1 & \text{if } [C] \in \bar{\phi}(\bar{\mathcal{D}}_1), \\ 2 & \text{if } [C] \in \bar{\phi}(\bar{\mathcal{D}}_2) \cup \bar{\phi}(\bar{\mathcal{D}}_3). \end{cases}$$

Proof. If $\phi^{-1}(C) = \{(D, D_0), (D', D_0)\}$, then $\bar{\phi}^{-1}([C]) = \{[D, D_0], [D', D_0]\}$. Note that $[D, D_0] = [D', D_0]$ if and only if there exists an element $\sigma \in S_n$ such taht $D^\sigma = D'$ and $D_0^\sigma = D_0$. This is equivalent to $\operatorname{Aut} D_0 \not\subset \operatorname{Aut} D$, or to $C \in \phi(\mathcal{D}_1)$. □

3 Enumeration

In this section, we describe how we enumerated the equivalence classes of Type I codes of length 32. As we have shown in the previous section, the set $\bar{\mathcal{C}}$ of equivalence classes of Type I codes can be constructed from the set $\bar{\mathcal{D}}$ via the maping $\bar{\phi}$. The enumeration of the elements of $\bar{\mathcal{D}}$ can be done easily by Magma [2] using the known classification of Type II codes of length 32 available in an electronic form [8].

Every element of $\bar{\mathcal{D}}$ is of the form $[D, D_0]$, where D is one of the 85 representatives for the equivalence classes of Type II codes of length 32. Then, for a fixed Type II code D, we need to enumerate all elements of $\bar{\mathcal{D}}$ of the form $[D, D_0]$. Note that for two subcodes D_0, D_0' of codimension 1 in D, $[D, D_0] = [D, D_0']$ holds if and only if there exists an element $\sigma \in \operatorname{Aut} D$ such that $D_0^\sigma = D_0'$. This means that we need to decompose the set of codimension 1 subcodes of D containing $\mathbf{1}$ into $\operatorname{Aut} D$-orbits. Equivalently, we wish to decompose the set of even codes of dimension $n/2 + 1$ containing D into $\operatorname{Aut} D$-orbits. This set corresponds bijectively to the set of nonzero elements of P/D, where P denotes the even weight code $\langle \mathbf{1} \rangle^\perp$. Since $\operatorname{Aut} D$ leaves P invariant, and D is an $\operatorname{Aut} D$-submodule of P, the matrix representation of $\operatorname{Aut} D$ on the quotient module P/D can be constructed by Magma. We give here explicitly a magma program to enumerate a set of representatives (D, D_0) for $\bar{\mathcal{D}}$ when D is the extended quadratic residue code of length 32.

```
> D:=ExtendCode(QRCode(GF(2),31));
> A:=AutomorphismGroup(D);
> M:=PermutationModule(A,GF(2));
> E:=EvenWeightCode(32);
> EM,inc:=sub< M | VectorSpace(E) >;
> DM:=sub< M | VectorSpace(D) >;
```

```
> N,proj:=quo< EM | DM >;
> orbits:=Orbits(MatrixGroup(N));
> reps:={ Rep(o) : o in orbits } diff { VectorSpace(N)!0 };
> #reps;
15
```

This shows that, if we denote by XQR_{32} the extended quadratic residue code of length 32, the set

$$\{[D, D_0] \in \bar{\mathcal{D}} \mid D \cong XQR_{32}\}$$

has 15 elements. One can execute a similar program for each of the 85 Type II codes.

Our next task is to decompose the set $\bar{\mathcal{D}}$ into $\bar{\mathcal{D}}_1, \bar{\mathcal{D}}_2$ and $\bar{\mathcal{D}}_3$. This can also be achieved easily by Magma. Let $[D, D_0] \in \bar{\mathcal{D}}$. We first check if $\operatorname{Aut} D_0$ is a subgroup of $\operatorname{Aut} D$. If not, we have $[D, D_0] \in \bar{\mathcal{D}}_1$ by the definition. If $\operatorname{Aut} D_0 \subset \operatorname{Aut} D$, then we compute a set of representatives for $D_0^{\perp}/D_0$. It follows from Lemma 1 that there exists a unique element v among the four representatives, satisfying $v \notin D$ and $\operatorname{wt}(v) \equiv 0 \pmod 4$. Then $D' = \langle D_0, v\rangle$ is a Type II code. If $D \cong D'$, then we conclude $[D, D_0] \in \bar{\mathcal{D}}_3$, otherwise we have $[D, D_0] \in \bar{\mathcal{D}}_2$. This can be done if we use Magma to test equivalence of the two codes D and D'. Instead, we can also use known properties of Type II codes in order to distinguish inequivalent Type II codes. In fact, the equivalence classes of Type II codes of length 32 can be uniquely identifiable by the order of automorphism group and the number of codewords of weight 4. Thus, if we compute the order of the automorphism group and the number of codewords of weight 4 for D, D', we can decide whether D is equivalent to D' or not.

Let C_i be the i-th Type II code of length 32 in the classification given in [4]. In Table 2, we list the sizes of the sets

$$|\{[C_i, D_0] \in \bar{\mathcal{D}}_j \mid \min \phi(C_i, D_0) = k\}| \tag{2}$$

in its i-th row, j-th column. The numbers are given as triples for $\bar{\mathcal{D}}_1$, corresponding to the values $k = 2, 4, 6$. For $\bar{\mathcal{D}}_2$, they are given for $k = 4, 6, 8$. For $\bar{\mathcal{D}}_3$, they are given for $k = 4$ only. Note that the minimum weight of a Type I code of length 32 can be $2, 4, 6$ or 8. A Type I code with minimum weight 2 exists only in $\phi(\mathcal{D}_1)$, and a Type I code with minimum weight 8 exists only in $\phi(\mathcal{D}_2)$. Indeed, if a Type I code C has minimum weight 2, then the transposition τ switching the two coordinates in the support of an element of weight 2 is an automorphism of C. It is easy to see that τ exchanges the two doubly-even neighbors D, D' of C. Thus $\operatorname{Aut} D_0 = \operatorname{Aut} C \not\subset \operatorname{Aut} D$, where

$D_0 = D \cap C$ is the doubly-even subcode of C. Hence $(D, D_0) \in \mathcal{D}_1$, and $C = \phi(D, D_0) \in \phi(\mathcal{D}_1)$. If a Type I code C has minimum weight 8, then it is shown in [5] that C has a doubly-even neighbor D with minimum weight 8, and that the shadow $(C \cap D)^\perp - C$ has minimum weight 4. This implies that the other doubly-even neighbor D' of C has minimum weight 4, hence $D \not\cong D'$. Thus $(D, D_0) \in \mathcal{D}_2$, and $C = \phi(D, D_0) \in \phi(\mathcal{D}_2)$.

We did not find any Type I code with minimum weight 6 in $\phi(\mathcal{D}_3)$. Probably there is a reason for that.

By Lemma 4, we can read off the number of equivalence classes of Type I codes from Table 2. Indeed, by Lemma 4, each equivalence class in $\bar{\phi}(\bar{\mathcal{D}}_2) \cup \bar{\phi}(\bar{\mathcal{D}}_3)$ is counted twice in Table 2. Thus the total number of equivalence classes of Type I codes of length 32 with minimum weight 4 is

$$480 + \frac{3824}{2} + \frac{20}{2} = 2402,$$

while the total number of equivalence classes of Type I codes of length 32 with minimum weight 6 is

$$17 + \frac{114}{2} = 74.$$

Therefore, we have established Theorem 1.

We note that Type I codes of length 32 with minimum weight 8 have already been classified in [5]. Also, Type I codes of length 32 with minimum weight 2 correspond bijectively to self-dual codes of length 30, and the number 731 has been given in [4]. Type I codes in $\bar{\phi}(\bar{\mathcal{D}}_2)$ can be found without repetition if we observe that the restriction of $\bar{\phi}$ to the set

$$\bigcup_{i=1}^{84} \{[C_i, D_0] \in \bar{\mathcal{D}}_2 \mid [C_i, D_0] = [C_j, D_0'] \text{ for some } j > i\}$$

is injective. Type I codes in $\bar{\phi}(\bar{\mathcal{D}}_3)$ are represented by two elements of $\bar{\mathcal{D}}_3$, so we need to use Magma to classify them into equivalent pairs. We do not give here explicit description of the Type I codes we have classified, but we note that we have checked the mass formula (1).

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Table 2: Type I neighbours

	$\mathcal{D}_1$			$\mathcal{D}_2$			$\mathcal{D}_3$		$\mathcal{D}_1$			$\mathcal{D}_2$			$\mathcal{D}_3$
min	2	4	6	4	6	8	6	min	2	4	6	4	6	8	6
C1	2	0	0	3	0	0	0	C44	4	0	0	11	0	0	0
C2	4	0	0	8	0	0	0	C45	6	3	0	21	0	0	0
C3	5	0	0	12	0	0	0	C46	13	6	0	56	0	0	0
C4	5	0	0	12	0	0	0	C47	12	9	0	46	0	0	0
C5	3	0	0	9	0	0	0	C48	7	3	0	25	0	0	0
C6	5	0	0	14	0	0	0	C49	6	2	0	13	0	0	0
C7	6	1	0	20	0	0	0	C50	6	2	0	30	0	0	0
C8	11	2	0	26	0	0	0	C51	6	0	0	27	0	0	0
C9	6	1	0	20	0	0	0	C52	13	5	0	71	0	0	0
C10	5	0	0	17	0	0	0	C53	8	2	0	37	0	0	0
C11	6	0	0	22	0	0	0	C54	12	11	0	81	0	0	0
C12	11	3	0	41	0	0	0	C55	15	17	0	106	0	0	0
C13	9	1	0	29	0	0	0	C56	16	13	0	113	0	0	0
C14	9	2	0	31	0	0	0	C57	13	8	0	69	0	0	0
C15	5	2	0	18	0	0	0	C58	18	17	0	126	0	0	0
C16	5	0	0	14	0	0	0	C59	14	12	0	95	0	0	0
C17	7	1	0	25	0	0	0	C60	6	3	0	24	0	0	0
C18	8	2	0	23	0	0	0	C61	19	20	0	152	0	0	2
C19	17	6	0	62	0	0	0	C62	21	29	0	162	0	0	0
C20	15	4	0	65	0	0	0	C63	18	23	0	142	0	0	0
C21	16	8	0	70	0	0	0	C64	12	15	0	77	0	0	0
C22	9	4	0	31	0	0	0	C65	14	15	0	67	0	0	0
C23	4	1	0	8	0	0	0	C66	6	4	0	22	0	0	0
C24	2	0	0	6	0	0	0	C67	2	0	0	9	0	1	0
C25	5	1	0	21	0	0	0	C68	3	0	0	18	0	1	0
C26	5	1	0	17	0	0	0	C69	2	1	0	13	0	1	0
C27	4	1	0	12	0	0	0	C70	7	4	0	53	1	0	0
C28	3	0	0	4	0	0	0	C71	9	13	0	73	1	0	0
C29	3	0	0	16	0	0	0	C72	10	8	0	66	1	0	2
C30	4	0	0	22	0	0	0	C73	6	14	0	70	1	0	0
C31	6	2	0	34	0	0	0	C74	7	3	0	35	4	0	0
C32	10	2	0	31	0	0	0	C75	14	27	0	140	4	0	6
C33	14	5	0	62	0	0	0	C76	13	27	0	115	5	0	4
C34	8	3	0	44	0	0	0	C77	6	4	1	25	7	0	0
C35	18	8	0	77	0	0	0	C78	8	17	1	66	10	0	0
C36	7	1	0	37	0	0	0	C79	8	9	1	54	8	0	0
C37	22	12	0	118	0	0	2	C80	8	13	4	44	16	0	4
C38	20	13	0	122	0	0	0	C81	1	0	3	0	11	0	0
C39	17	12	0	101	0	0	0	C82	1	0	0	0	3	1	0
C40	7	5	0	42	0	0	0	C83	2	0	0	0	7	1	0
C41	12	9	0	68	0	0	0	C84	2	0	2	0	15	1	0
C42	9	6	0	40	0	0	0	C85	3	0	5	0	20	0	0
C43	5	2	0	16	0	0	0	total	731	480	17	3824	114	6	20