

東北大学大学院情報科学研究科
純粋・応用数学研究センター

情報数理談話会のお知らせ

日 時： 2015年12月22日 16:00より17:00まで
(会場にお茶を用意しております)

場 所： 東北大学大学院情報科学研究科棟 2階中講義室

講演者： Minwon Na 氏 (東北大学大学院情報科学研究科)

題 目： Three algorithms to construct semistandard Young tableaux, and a generalization of Knuth's formula for the number of skew tableaux

備 考： この情報数理談話会は課程博士予備審査会を兼ねています

[概要] Kostka numbers give the number of semistandard tableaux of given shape and weight, and they play a fundamental role in representation theory of symmetric groups. Much work has been done on the problem of computing Kostka numbers, which is in general difficult. There is a recurrence formula for Kostka numbers, but we have no explicit formula for Kostka numbers.

First of all, we give three algorithms to construct a semistandard tableau of given shape and weight, where weight is a composition which is not necessarily a partition. We also introduce a natural partial order on the set of semistandard tableaux, and show that the set of semistandard tableaux of given shape and weight has a unique greatest element and a unique least element. Two of our algorithms give each of these elements.

After that, we take an elementary approach to derive a generalization of Knuth's formula using Lassalle's explicit formula. In particular, we give a formula for the Kostka numbers of a shape $\mu \vdash n$ and weight $(m, 1^{n-m})$ for $m = 3, 4$.

Finally, since the Kostka number is defined to be the cardinality of the set of all semistandard tableaux of given shape and weight, it is natural to expect a bijective proof of Vershik's relations for the Kostka numbers. We describe a bijective proof of Vershik's relations for the Kostka numbers using row bumping and reverse row bumping algorithms. In order to prove it, we turn the bumping algorithm for semistandard tableaux to a sequence called a bumping route. We also define a reverse bumping route by reversing the algorithm. These sequences are used to give a bijective proof of Vershik's relations for the Kostka numbers.