

東北大学大学院情報科学研究科
純粹・応用数学研究センター

情報数理談話会のお知らせ

日 時： 2019 年 12 月 17 日 (火) 15:00 より 16:00 まで

(会場にお茶を用意しております)

場 所： 東北大学大学院情報科学研究科棟 2 階大講義室

講演者： Chen Gangqiang (陳 剛強) 氏 (東北大学大学院情報科学研究科)

題 目： On the second derivative of bounded analytic functions

備 考： この情報数理談話会は課程博士予備審査会を兼ねています

[概 要] For c in the complex plane \mathbb{C} and $r > 0$, let $\mathbb{D}(c, r) = \{z \in \mathbb{C} : |z - c| < r\}$ and $\overline{\mathbb{D}}(c, r) = \{z \in \mathbb{C} : |z - c| \leq r\}$. In particular we denote the open unit disk $\mathbb{D}(0, 1)$ by \mathbb{D} . Assume that a point $z \in \mathbb{D}$ and f is an analytic self-map of \mathbb{D} fixing 0. Schwarz's lemma asserts that $|f(z)| \leq |z|$, and Dieudonné's lemma derives an inequality about the derivative $f'(z)$, $|f'(z)| \leq \min\{1, (1 + |z|^2)/(4|z|(1 - |z|^2))\}$, which is best possible for each value of $|z|$. In this talk, we shall obtain a sharp upper bound for the second derivative $f''(z)$ depending only on $|z|$.

Furthermore, assume that $w \in \mathbb{D}$ with the modulus of z greater than that of w . Schwarz's lemma shows that $\{f(z)\} = \overline{\mathbb{D}}(0, |z|)$. Dieudonné's lemma asserts that

$$\{f'(z) : f(z) = w\} = \overline{\mathbb{D}}\left(\frac{w}{z}, \frac{|z|^2 - |w|^2}{|z|(1 - |z|^2)}\right).$$

We shall determine the variability region $\{f''(z) : f(z) = w\}$ when f ranges over the class of all analytic self-maps of unit disk fixing 0. We also graphically illustrate our main result by using Mathematica.