Multivariate Krawtchouk polynomials: construction and associated quantization

Philip Feinsilver

Southern Illinois University, USA

Abstract: Multivariate Krawtchouk polynomials arise via symmetric tensor powers by specification of an orthogonal matrix and two positive definite diagonal matrices. They are orthogonal polynomials with respect to multinomial distributions. After detailing their construc- tion, they are recognized as "Bernoulli systems" — yielding a family of commuting self-adjoint operators associated to systems of partial differential equations of Riccati type.

Spherical design and its generalization: recent results and open problems

Djoko Suprijanto

Department of Mathematics, Institut Teknologi Bandung, Indonesia

Abstract: One important object in combinatorial mathematics is a *combinatorial block t-design*, which was introduced for the first time by statisticians. In 1977, Delsarte, Goethals, and Seidel introduced a *spherical analogue* of the above concept called *spherical t-design.*

Spherical designs may be regarded as a finite set approximating a unit sphere with respect to an integral of polynomial functions. Formally, a *spherical t-design* on a unit sphere $S^d \subseteq \mathbb{R}^{d+1}$ is a finite set $X \subseteq S^d$ such that the *Chebyshev-type quadrature formula*

$$\frac{1}{\sigma(S^d)} \int_{S^d} f(x) d\sigma(x) \approx \frac{1}{|X|} \sum_{x \in X} f(x)$$

is exact for all polynomials $f(x) = f(x_0, x_1, x_2, \ldots, x_d)$ of degree at most t.

In this talk, we introduce basic concepts of spherical t-designs and review recent results including several methods to construct them. Moreover, we discuss some directions of generalization of this concept. We mention also several open problems in this area.

I have to admit that the content of my talk is not new for experts. The purpose of my talk is to attract amateur mathematicians, in particular amateur combinatorialists, to this (hopefully) interesting concept from my personal viewpoint.

Distance in graphs: two old and one new problems

Rinovia Simanjuntak

Department of Mathematics, Institut Teknologi Bandung, Indonesia

Abstract: The distance between two vertices in a graph is the length of the shortest path connecting the vertices. Determining the distance between a pair of vertices in a graph is known to be solvable in polynomial time [Dijkstra (1959), Floyd and Warshall (1962)]. Since then, many distance-related problems have arisen through the years. I will present three problems on which I am currently working. In **degree/diameter problem** we maximize the order of a graph given its maximum degree and diameter. By **metric dimension** the position of a vertex in a graph is uniquely determined using distances. And with **distance-magic labeling** we generalize the well-known magic labeling notion through distances. The *diameter* of a graph G is the greatest distance between two vertices in G. Con- sidering three parameters in a graph: order, degree, and diameter, we can optimize one parameter while fixing the other two parameters. Among three optimization pos- sibilities, maximizing the order of the graph given its maximum degree and diameter is one problem that has attracted many researchers since first introduced by Moore, Hoffman, and Singleton in 1960. A natural upper bound for the maximum order is known due to Moore and a graph whose order attaining such a bound is then called a Moore graph. Unfortunately, or maybe fortunately for the sake of further research, there only exist very few Moore graphs. We have contributed several results: proving non-existence of graphs of maximum degree 4 with order close to the Moore bound and thus improving the upper bound for graphs with that particular degree; and determin- ing "good" bounds for more restricted versions of the degree/diameter problem (i.e., graphs embedable in fixed surfaces and bipartite Cayley graphs). Motivated by two real world applications: assigning computer memory network and robotic navigation, the notion of metric dimension was introduced separately by Slater (1975) and Harary and Melter (1976). They called a set of vertices S *resolves* a graph G if every vertex is uniquely determined by its vector of distances to the vertices in S. The *metric dimension of G, dim(G)*, is the minimum cardinality of a resolving set of G. Determining the metric dimension of an arbitrary graph has been proved to be NP-hard [Garey and Johnson (1979)] and so research in this area are then constrained towards: characterizing graphs with particular metric dimensions, determining metric dimensions of particular graphs, and constructing algorithm that best approximate metric dimensions. Our research has contributed results in the first two topics. Vilfred (1994) and Miller, Rodger, Simanjuntak (2003) defined *distance-magic la- beling* of a graph G as labeling of vertices in G by consecutive integers from 1 up to the order of G such that the summation of all labels of a vertex's neighbors (i.e., vertices of distance one) is independent of the choice of vertex. In a way, this labeling can be viewed as a natural extension of previously known graph labelings: the magic labeling [Sedlacek (1963), Kotzig and Rosa (1970)] and

radio labeling (which is distance-based) [Griggs and Yeh (1992)]. Recently, O'Neal and Slater (2012) has generalized the notion of distance-magic labeling by considering the summation for labels of all vertices with particular distances to the vertex under consideration, not only its neighbors. A short historical account, known techniques, recent results, and open questions of the three afore-mentioned problems will be presented.

NLS on graphs. (Few) Results and (many) problems.

Riccardo Adami

Politecnico di Torino (Department of Mathematical Sciences), Corso Duca degli Abruzzi 24, 10129 Torino, Italy

Abstract: Schroedinger equation on graphs is nowadays an established field of research, connected with applications to nanotechnologies and quantum devices. It has been recently shown that also nonlinear Schroedinger equation on ramified structures proves interesnting both from the physical and from the analytical side. We give a survey on recent results, with particular emphasis on the problem of the stability of stationary states, and on open problems. This is a joint research project with C. Cacciapuoti (Bonn), D. Finco (Rome), and D. Noja (Milan).

A characterization of ellipsoids as uniformly dense sets with respect to a family of convex bodies

Rolando Magnanini

Dipartimento di Matematica e Informatica "U. Dini", Università di Firenze, viale Morgagni 67/A, 50134 Firenze, Italy.
magnanin@math.unifi.it

Abstract: Let K be an N-dimensional convex body containing the origin. We say that a Lebesgue-measurable set $G \subset \mathbb{R}^N$ of finite positive measure is *uniformly K-dense*, or *K-dense* for short, if, for every fixed $r > 0$, the measure of the sets $G \cap (x + rK)$ is a constant $c(r)$ for every $x \in \partial G$ (here, $x + rK$ denotes a translate of a dilate of K). This property has to do with time-invariant level surfaces of solutions of certain parabolic partial differential equations.

For $N = 2$, we prove that, if G is K-dense, then both G and K are the same ellipse up to translations and dilations. By a different proof, this result extends to full generality (i.e. with no regularity assumptions on K or G) a theorem by Amar, Berrone and Gianni.

For general N, we first show that, when K is centrally symmetric, $K = G - G$ up to homotheties, i.e. K is the Minkowski sum of G and $-G$. When also G is centrally symmetric, we prove that both G and K must be the same ellipsoid up to translations and dilations.

The ingredients of the proof are essentially two: an asymptotic formula for the measure of $G \cap (x + rK)$ for "large" values of r and a characterization of ellipsoids due to Petty.

This talk is based on work done in collaboration with Michele Marini (Scuola Normale Superiore Pisa).

On the hyperbolic metric on a Riemann surface with conical singularities

Toshiyuki Sugawa

RCPAM, Graduate School of Information Sciences, Tohoku University

Abstract: It is a classical result that an analytically finite Riemann surface (i.e., a compact Riemann surface minus finitely many points) carries a canonical complete Riemannian metric of constant Gaussian curvature -1 with prescribed conical singularities as long as the virtual hyperbolic area is positive. By using a potential-theoretic method due to Heins, we will show existence of such a metric even when the surface is analytically infinite. We will also give some background, concrete estimates and related problems.