François Alouges
Periodic homogenization of PDEs
Course syllabus

This series of lectures is devoted to introduce the basic concepts of homogenization of periodically microstructured materials. Roughly speaking the aim of the topic is to understand the material as an averaged material in which the average has to be explicited in the right way. When materials are modelled with a partial differential equation (PDE), the problem becomes to mathematically understanding the limiting process as the size of the microstructure tends to 0, of the solution of a PDE with fast varying coefficients.

Among the classical theoretical methods used to study this kind of problems, we focus on the multiscale expansion method and the 2-scale convergence. Both give results of different flavours, heuristic or rigorous ones, and happen to be very complementary.

As a matter of fact, the multiscale expansion method works by assuming an ansatz for the solution for which each coefficients are sought one after the other. The derivation is done under « reasonable » assumptions for the existence of such an expansion. Although very heuristic, the method turns out to give the right solutions.

On the other hand, the 2-scale convergence theory of N'Guetseng and Allaire allows for a complete and rigorous approach.

We also plan to provide the students that follow the course with further aspects on numerical aspects together with Gamma convergence arguments that are linked with the subject.

The course is self-contained, however a basic knowledge of the analysis of PDEs is required to follow this course.

www.math.is.tohoku.ac.jp/~gaseminar/ss2015.html