Resolvable Steiner designs and maximal arcs in projective planes

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Let $D = \{X, \mathcal{B}\}$ be a Steiner 2-$(v, k, 1)$ design with point set X, collection of blocks $\mathcal{B}$, and let v be a multiple of k, $v = nk$. A *parallel class* is a set of $v/k = n$ pairwise disjoint blocks that partition X, and a *resolution* is a partition R of $\mathcal{B}$ into $r = (v-1)/(k-1)$ disjoint parallel classes. A design is *resolvable* if it admits a resolution. Two resolutions R_1, R_2,

$$R_1 = P_1^{(1)} \cup P_2^{(1)} \cup \cdots P_r^{(1)}, \ R_2 = P_1^{(2)} \cup P_2^{(2)} \cup \cdots P_r^{(2)}$$

are called *compatible* [1] if they share one parallel class, $P_i^{(1)} = P_j^{(2)}$, and $|P_{i'}^{(1)} \cap P_{j'}^{(2)}| \leq 1$ for $(i', j') \neq (i, j)$.

A maximal $(q(k-1)+k, k)$-arc in a finite projective plane of order $q = sk$ is a set A of $q(k-1)+k$ points such that every line is either disjoint form A, or meets A in exactly k points.

An upper bound on the maximum number of mutually compatible resolutions of a resolvable 2-$(nk, k, 1)$ design D was proved in [1]. The bound is attainable if and only if D is embeddable as a maximal $(kq-q+k, k)$-arc in a projective plane of order $q = (v-k)/(k-1)$.

The maximal sets of mutually compatible resolutions of 2-$(52, 4, 1)$ designs associated with known and newly found maximal $(52, 4)$-arcs in projective planes of order 16 were computed recently in [2]. It was shown that some 2-$(52, 4, 1)$ designs can be embedded as maximal arcs in nonisomorphic planes. This phenomenon establishes new links between the known planes of order 16, and motivates the problem of completing the classification of maximal $(52, 4)$-arcs, initiated in [3].

References

[1] Vladimir D. Tonchev, On resolvable Steiner 2-designs and maximal arcs in projective planes, *Designs, Codes and Cryptography* **84**, No. 1 - 2 (2017), 165 - 172.

[2] M. Gezek, T. Wagner and V. D. Tonchev, Maximal arcs in projective planes of order 16 and related designs, *Advances in Geometry*, to appear.

[3] N. Hamilton, S. Stoichev, and V. D. Tonchev, Maximal arcs and disjoint maximal arcs in projective planes of order 16, *J. Geometry* **67** (2000), 117-126.