Counting Steiner triple systems of given 2-rank and 3-rank

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This lecture is based on joint work with Dieter Jungnickel [3], [4].

By a famous result of Doyen, Hubaut and Vandensavel [2], the 2-rank of the incidence matrix of a Steiner triple system on $2^n - 1$ points is at least $2^n - 1 - n$, and equality holds only for the classical design of points and lines in the binary projective geometry $PG(n-1, 2)$. It follows from results of Assmus [1] that, given any integer t with $1 \leq t \leq n-1$, there is a binary linear code $C_{n,t}$ of length $2^n - 1$ and dimension $2^n - 1 - n + t$ that contains representatives of all isomorphism classes of $STS(2^n - 1)$ of 2-rank at most $2^n - 1 - n + t$. Using a mixture of coding theoretic, geometric, design theoretic and combinatorial arguments, we prove a general formula for the number of distinct $STS(2^n - 1)$ having 2-rank at most $2^n - 1 - n + t$ contained in this code. This generalizes previously known results, which only cover the cases $t \leq 3$ (Tonchev [5], V. Zinoviev and D. Zinoviev [7], D. Zinoviev [6]). Finally, using our recent systematic study of the ternary linear codes of Steiner triple systems [4], we obtain analogous results for the ternary case, and a formula for the number of $STS(3^n)$ having 3-rank at most $3^n - 1 - n + t$.

References

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