Introductory Lectures on Quantum Probability

量子確率論への招待

Dates: Aug 29 – 30, 2011
Venue: Mid-Campus Open Laboratory Building No.2
(Research Center for Integrative Mathematics)
Research Institute for Electronic Science, Hokkaido University
http://www.es.hokudai.ac.jp/english/
Map: http://www.math.jst.go.jp/office/map.pdf
北海道大学電子科学研究所
中央キャンパス総合研究棟２号館５階講義室

量子確率論のいくつかの話題について入門講義を企画しました。非線形数学・パターン形成・生命系の数学・ネットワークなどの研究者とのコラボを遠望しています。どなたでも興味のある方はご自由にご参加ください。

Time Table

	13:30-14:30	14:40-15:40	15:50-16:50
Aug 29 (Mon)	Nobuaki Obata	Hayato Saigo	Un Cig Ji
Aug 30 (Tue)	Un Cig Ji	Jaeseong Heo	Hayato Saigo Takahiro Hasebe

講義題目
尾畑伸明（東北大学大学院情報科学研究科）
「量子確率論の広がり」
西郷甲矢人（長浜バイオ大学）・長谷部高広（京都大学）
「Noncommutative independence and generalized cumulants」
Un Cig Ji (Chungbuk National University, Korea)
「Quantum white noise approach to quantum stochastic calculus」
Jaeseong Heo (Hanyang University, Korea)
「Von Neumann algebras and free probability」

講義概要

量子確率論の広がり

尾畑伸明（東北大学大学院情報科学研究科）

量子確率論のルーツはフォンノイマンの有名な著書「量子力学の数学的基礎」(1932)に遡るが、1980 年代以降、量子伊藤解析の流れと自由確率論に代表される非可換解析の流れに沿って大きく発展している比較的新しい研究分野である。本講義では、量子確率論の基本的な概念と最近の研究の広がりについて概観したい。

[1] K. R. Parthasarathy: An introduction to quantum stochastic calculus, Springer Monographs in Mathematics, Birkhäuser, 1992.
[2] P. A. Meyer: Quantum probability for probabilists, Lecture Notes in Mathematics Vol. 1538, Springer, 1995.
[3] A. Hora and N. Obata: Quantum probability and spectral analysis of graphs, Springer, 2007.
[4] 明出伊類似・尾畑伸明：量子確率論の基礎, 牧野書店, 2003.

Noncommutative independence and generalized cumulants

Hayato Saigo and Takahiro Hasebe

Noncommutative probability is such an algebraic framework extending probabilistic notions that its scope is to cover various areas of mathematics and sciences related to quantum theory. While usual probability theory mainly treats random variables commuting with each other, in quantum theory it is essential to deal with noncommutative random variables which we call observables. Then states are nothing but the expectation functionals for noncommutative random variables.

One of the central issues of interest here is the extension of the notion of "independence," which is enriched by much "variety" due to noncommutativity. For each "independence," we can discuss the associated limit theorems. For example, "monotone independence (Muraki, Lu)" is a unique one as an "asymmetric" kind characterized by certain naturality conditions, in the sense that monotone independence of "X from Y" does not imply that of "Y from X." The monotone version of central limit theorem is first given by Muraki, with its "limit distribution" not being the Gaussian but the "arcsine law."

What is most relevant to the unification (and extension) of such types of the limit theorems is the notion of "generalized cumulants." By the use of generalized cumulants, we have obtained quite simple proofs for the limit theorems in the unified manner. Moreover, the combinatorial representation of the relation between moments and generalized cumulants are obtained.

Von Neumann algebras and free probability

Jaeseong Heo

Motivated by quantum mechanics and group representation theory, John von Neumann introduced certain algebras of bounded operators on a Hilbert space, the so-called von Neumann algebras. In his double commutant theorem he showed that these algebras can be characterized either in purely topological or in purely algebraic terms that has numerous beautiful and deep consequences. The theory of operator algebras has many fruitful interrelations and fruitful connections with other areas of mathematics and physics, in particular, with low dimensional topology, knot theory, quantum groups, statistical mechanics and quantum field theory.

Since Wigner's work in 1955, random matrices have been a very important tool in mathematical physics and probability theory. In functional analysis, random matrices have been used to construct Banach spaces with surprising properties. Voiculescu in the early 1990's found that large random matrices provide an asymptotic model of free probability theory. He introduced the free entropy that goes with free independence, as the free analogue of the entropy quantity in Shannon's information theory.

We will mainly introduce von Neumann algebras and noncomutative probability theory and discuss some problems in operator algebras.

Quantum white noise approach to quantum stochastic calculus

Un Cig Ji

Motivated by the white noise theory and quantum stochastic calculus, we introduce quantum white noise theory [Ji-Obata02] based on a Gelfand triple:

$$(E) \subset \Gamma\big(L^2(R, dt)\big) \subset (E)^*,$$

which is a mathematical framework of infinite dimensional distribution theory [Obata94]. A pair $\{a_t, a_t^* \,; t \in R\} \subset L((E),(E)^*)$ of pointwisely defined annihilation operator a_t and creation operator a_t^* is called a *quantum white noise* [Ji-Obata02]. Based on the quantum white noise $\{a_t , a_t^*\}$, we study white noise operators in $L((E),(E)^*)$ and then every white noise operator admits the *Fock expansion* [Obata94] which is a series expansion in terms of quantum white noise $\{a_t , a_t^*\}$ which means that every white noise operator can be considered as a function of quantum white noise $\{a_t , a_t^*\}$. Then it is natural to consider a kind of functional derivatives of white noise operators with respect to the annihilation and creation operators. The derivatives are called
the *quantum white noise derivatives* [Ji-Obata09a]. Based on the quantum white noise derivatives we introduce the *quantum stochastic gradients* [Ji-Obata09c] and then we study the *quantum stochastic integrals* as the adjoint operators of the quantum stochastic gradients.

During the discussions, we study the analytic characterization of white noise functionals and white noise operators which is one of the fundamental results in the classical and quantum white noise theories. We also discuss some applications of quantum white noise derivatives.

For our purpose, we focus mostly on the backgrounds and motivations of the theories and discuss several applications. Then we hope that we can find several interesting research topics for future collaborations between participants.

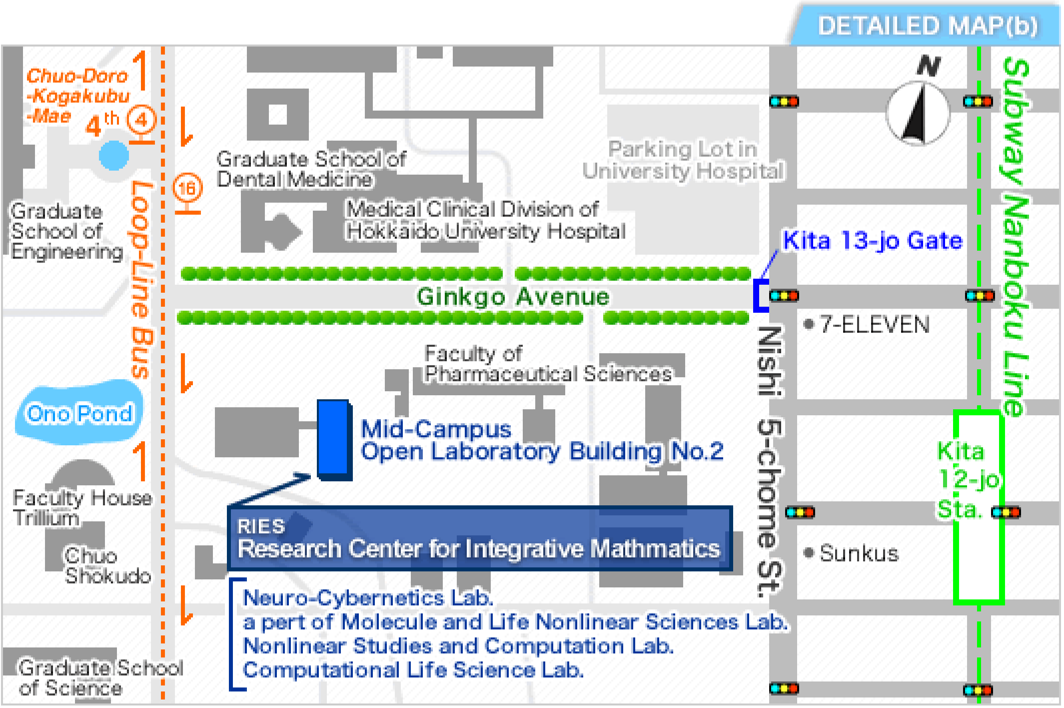
[Ji-Obata02] U. C. Ji and N. Obata: Quantum white noise calculus, in "Non-Commutativity, Infinite-Dimensionality and Probability at the Crossroads (N. Obata, T. Matsui and A. Hora, Eds.)," pp.143-191, World Scientific, 2002.

[Ji-Obata09a] U. C. Ji and N. Obata: Annihilation-derivative, creation-derivative and representation of quantum martingales, Commun. Math. Phys. 286 (2009), 751-775.

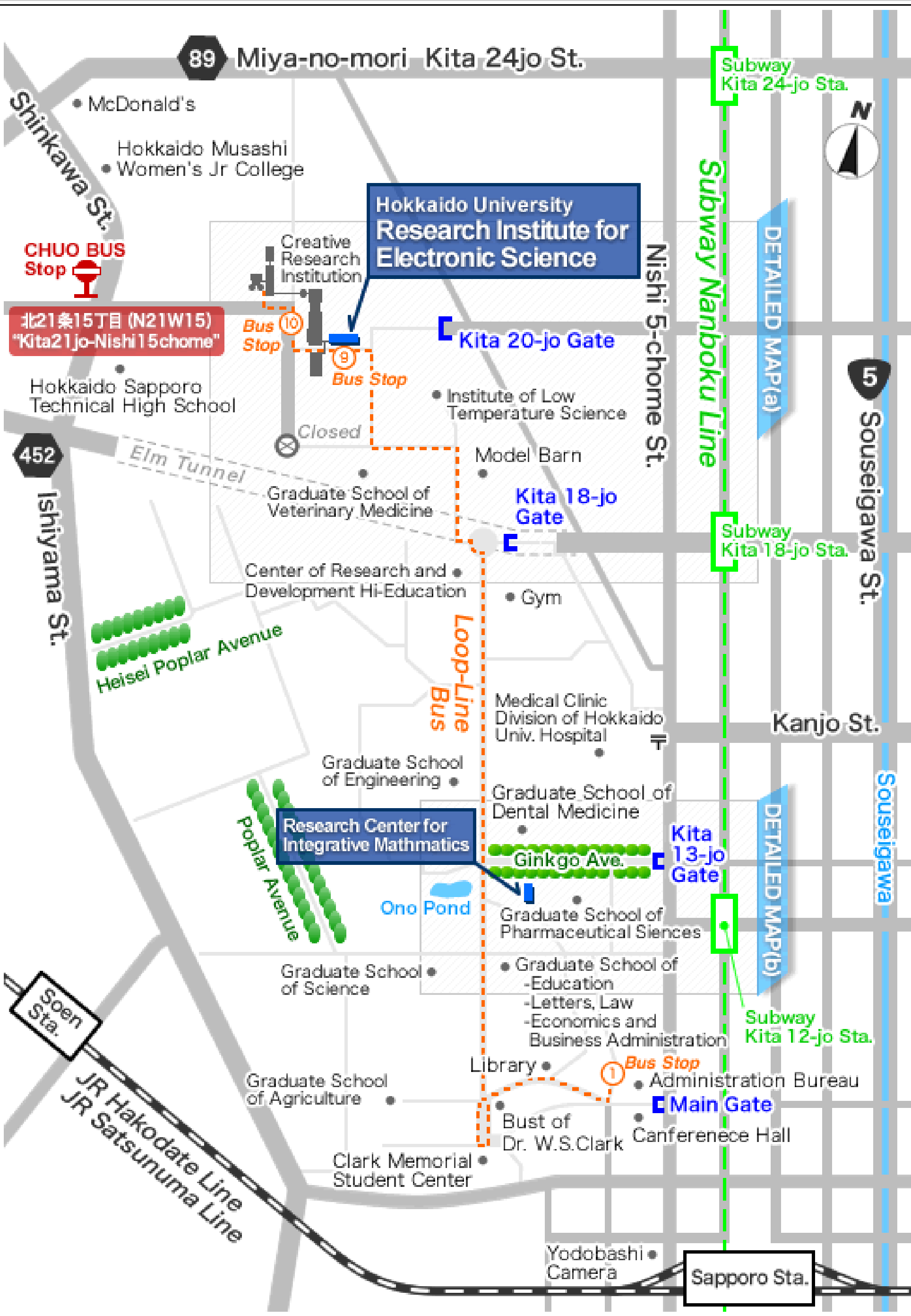
[Ji-Obata09c] U. C. Ji and N. Obata: Quantum stochastic gradients, Interdiscip. Inform. Sci. 14 (2009), 345-359.

[Ji-Obata10] U. C. Ji and N. Obata: Implementation problem for the canonical commutation relation in terms of quantum white noise derivatives, J. Math. Phys. 51 (2010), 123507.

[Obata94] N. Obata: "White Noise Calculus and Fock Space," Lecture Notes in Math. Vol. 1577, Springer, 1994.



DETAILED MAP(b)
N
Chuo-Doro -Kogakubu -Mae
4th
4
16
Graduate School of Dental Medicine
Parking Lot in University Hospital
Graduate School of Engineering
Medical Clinical Division of Hokkaido University Hospital
Kita 13-jo Gate
Subway Nanboku Line
Loop-Line Bus
Ginkgo Avenue
7-ELEVEN
Faculty of Pharmaceutical Sciences
Ono Pond
Mid-Campus Open Laboratory Building No.2
Kita 12-jo Sta.
Faculty House Trillium
RIES
Research Center for Integrative Mathmatics
Nishi 5-chome St.
Sunkus
Chuo Shokudo
Neuro-Cybernetics Lab.
a pert of Molecule and Life Nonlinear Sciences Lab.
Nonlinear Studies and Computation Lab.
Computational Life Science Lab.
Graduate School of Science



89
Miya-no-mori Kita 24jo St.
Subway Kita 24-jo Sta.
McDonald's
Shinkawa St.
Hokkaido Musashi Women's Jr College
N
Hokkaido University
Research Institute for Electronic Science
Creative Research Institution
CHUO BUS Stop
北21条15丁目 (N21W15)
"Kita21jo-Nishi15chome"
Bus Stop
10
9
Bus Stop
Kita 20-jo Gate
Nishi 5-chome St.
Subway Nanboku Line
DETAILED MAP(a)
5
Souseigawa St.
Hokkaido Sapporo Technical High School
Institute of Low Temperature Science
Closed
452
Elm Tunnel
Model Barn
Graduate School of Veterinary Medicine
Kita 18-jo Gate
Subway Kita 18-jo Sta.
Ishiyama St.
Center of Research and Development Hi-Education
Gym
Heisei Poplar Avenue
Loop-Line Bus
Medical Clinic Division of Hokkaido Univ. Hospital
Kanjo St.
Graduate School of Engineering
Graduate School of Dental Medicine
Souseigawa
Research Center for Integrative Mathmatics
Kita 13-jo Gate
Ginkgo Ave.
DETAILED MAP(b)
Poplar Avenue
Ono Pond
Graduate School of Pharmaceutical Siences
Graduate School of Science
Graduate School of -Education -Letters, Law -Economics and Business Administration
Subway Kita 12-jo Sta.
Soen Sta.
Library
Bus Stop
1
Administration Bureau
Graduate School of Agriculture
Main Gate
Bust of Dr. W.S.Clark
Canference Hall
JR Hakodate Line
JR Satsunuma Line
Clark Memorial Student Center
Yodobashi Camera
Sapporo Sta.