Sapporo Workshop on Non-Commutative Analysis and Applications to Complex Phenomena

Boys, be ambitious!

September 1 (Thu) – 3 (Sat), 2011

Department of Mathematics

Hokkaido University

PROGRAM

September 1 (Thu)

13:30-14:20 Hun Hee Lee (Chungbuk National University, Korea)
Graph interpretation of q-gaussians

14:30-15:20 Jaeseong Heo (Hanyang University, Korea)
Bounded homomorphisms of C-algebras*

15:30-16:20 Takahiro Hasebe (Kyoto University)
On a class of explicit Cauchy-Stieltjes transforms related to monotone stable and free Poisson laws

16:30-17:20 Hiroaki Yoshida (Ochanomizu University)
The free Poisson limit by the free Fisher information distance

September 2 (Fri)

09:40-10:30 Fumio Hiai (Tohoku University)
Higher order extension of Löwner's theory: Operator k-tone functions
(joint work with Uwe Franz and Eric Ricard)

10:40-11:30 Abdessatar Barhoumi (Sousse University, Tunisia)
Quantum Euler operator and associated heat equation

11:40-12:30 Asao Arai (Hokkaido University)
Hilbert space representations of a non-commutative algebra indexed by a real anti-symmetric matrix

14:00-14:50 Un Cig Ji (Chungbuk National University, Korea)
Quantum stochastic calculus associated with quadratic quantum white noises

15:00-15:50 Hideo Kubo (Tohoku University)
Asymptotic behavior for nonlinear Dirac equation with small initial data

16:00-16:50 Sangho Kum (Chungbuk National University, Korea)
The resolvent average on symmetric cones

September 3 (Sat)

10:00-10:50 Hyun Jae Yoo (Hankyong National University, Korea)
Glauber dynamics on the cycles: A quantum probabilistic view

11:00-11:50 Akihito Hora (Nagoya University)
Harmonic functions on branching networks for some big groups

13:30-14:20 Hayato Saigo (Nagahama Institute of Bio-Science and Technology)
Noncommutative umbral calculus

14:30-15:20 Etsuo Segawa (University of Tokyo)
Quantum probabilistic approach to quantum walks on spidernets

15:30-16:20 Norio Konno (Yokohama National University)
Quantum walks on a path space

16:30 Closing

http://www.welcome.city.sapporo.jp/photolibrary/

ABSTRACTS

Hilbert space representations of a non-commutative algebra indexed by a real anti-symmetric matrix

Asao Arai (Department of Mathematics, Hokkaido University)
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We introduce a non-commutative algebra which includes non-commutative space-times and quantum phase spaces, and consider Hilbert space representations of it. We prove a uniqueness theorem on Weyl type representations of the non-commutative algebra.

Quantum Euler operator and associated heat equation

Abdessatar Barhoumi (Department of Mathematics, Sousse University, Tunisia)
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The main objective of this talk is to investigate the heat equation associated to the quantum Euler operator $\Delta_G^Q + N^Q$, where Δ_G^Q and N^Q stand for appropriate quantum counterparts of the Gross Laplacian and the conservation operator, respectively. It is shown that Δ_E^Q has an integral representation in terms of the quantum white noise derivatives $\{D_t^-, D_t^+ ; t \in \mathbf{R}\}$ as a kind of functional integral acting on nuclear algebra of white noise operators endowed with the convolution product. Similarly to the finite dimensional case, Δ_E^Q is shown to be a Wick derivation. Via this property, existence and uniqueness of the quantum Euler heat equation are proved.

On a class of explicit Cauchy-Stieltjes transforms related to monotone stable and free Poisson laws

Takahiro Hasebe (Graduate School of Science, Kyoto University)
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The Cauchy-Stieltjes transform of a probability measure cannot be calculated explicitly in generic cases. To investigate infinitely divisible distributions in free probability, we also have to calculate the inverse function of the Cauchy-Stieltjes transform. In this talk, we introduce a class of probability measures with explicit Cauchy-Stieltjes transforms related to monotone stable laws and free Poisson laws. (joint work with Octavio Arizmendi)

Bounded homomorphisms of C*-algebras

Jaeseong Heo (Department of Mathematics, Hanyang University, Korea)
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In this talk, I would like to discuss about the similarity question posed by Kadison. For this question, we consider bounded homomorphisms and bounded module maps. Our tools for this question are Grothendieck-Pisier-Haagerup inequality and averaging technique. Our strategy is to consider derivations on C*-algebras and to show the complete boundedness of bimodular

derivations.

Higher order extension of Löwner's theory: Operator k-tone functions

Fumio Hiai (Graduate School of Information Sciences, Tohoku University)
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The new notion of operator/matrix *k*-tone functions is introduced, which is a higher order extension of operator/matrix monotone and convex functions. Differential properties of matrix *k*-tone functions are shown. Characterizations, properties, and examples of operator *k*-tone functions are presented. In particular, integral representations of operator *k*-tone functions are given, generalizing familiar representations of operator monotone and convex functions. (joint work with Uwe Franz and Eric Ricard)

Harmonic functions on branching networks for some big groups

Akihito Hora (Graduate School of Mathematics, Nagoya University)
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Let *G* be an infinite wreath product group of a compact group. A dual object of *G* is given by a branching network *Y* like the Young graph. We compute the (minimal) Martin boundary of *Y* through an explicit expression of Martin kernels in terms of supersymmetric Schur functions. The minimal Martin boundary of *Y* has a bijective correspondence to the characters of *G*.

Quantum stochastic calculus associated with quadratic quantum white noises

Un Cig Ji (Department of Mathematics, Chungbuk National University, Korea)
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We study the quadratic quantum white noises as generalized quantum stochastic processes in white noise approach to quantum stochastic calculus. Then we define the quantum stochastic integrals with respect to the quadratic quantum white noises and study the quantum Itô formula for the quantum stochastic integrals.

Quantum walks on a path space

Norio Konno (Graduate School of Engineering, Yokohama National University)
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In this talk, we consider some properties of one-dimensional discrete- time quantum walks on a path space.

[1] N. Konno: Sojourn times of the Hadamard walk in one dimension, Quantum Information Processing (in press), arXiv:1102.5587
[2] S. Gudder: Discrete quantum processes, arXiv:1106.0019

Asymptotic behavior for nonlinear Dirac equation with small initial data

Hideo Kubo (Graduate School of Information Sciences, Tohoku University)
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In this talk, I'd like to discuss the solution of a nonlinear massless Dirac equation with small initial data in four dimensional space-time. The nonlinearity is supposed to vanish of second order at the origin. Then, the equation is of critical type, so that an interesting problem is to study how quadratic perturbations influence the lifespan When the slow time is small enough, one can solve the Cauchy problem around the solution to the unperturbed Dirac equation. In order to get a sharp lower bound of the lifespan, we need to construct an approximate solution which actually differs from the unperturbed solution when the slow time attains a certain value. This can be done by reducing a suitable nonlinear ordinary differential equation from the original equation.

The resolvent average on symmetric cones

Sangho Kum (Department of Mathematics, Chungbuk National University, Korea)
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Recently Bauschke *et al*. introduced a very interesting and new notion of proximal average in the context of convex analysis, and studied this subject systemically from various viewpoints. In addition, this new concept was applied to positive semidefinite matrices under the name of resolvent average, and basic properties of the resolvent average are successfully established by themselves from a totally different view and techniques of convex analysis rather than the classical matrix analysis. Inspired by their works and the well-known fact that the convex cone of positive definite matrices is a typical example of a symmetric cone (self-dual homogeneous convex cone), we study the resolvent average on symmetric cones, and derive corresponding results in a different manner based on a purely Jordan algebraic technique.

Graph interpretation of q-gaussians

Hun Hee Lee (Department of Mathematics, Chungbuk National University, Korea)
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Hora/Obata initiated a spectral analysis of graphs using quantum probabilistic approach. Their main focus was simple graphs i.e. undirected and unweighted and showed that several important distributions can be obtained as a limit distribution of well-known graphs. The examples include gaussian laws and semi-circular laws. In this talk I will explain that q-gaussians can be obtained as limit distributions of weighted and directed graphs, which gives a good reason for the directed and weighted version of Hora/Obata's theory.

Noncommutative umbral calculus

Hayato Saigo (Nagahama Institute of Bio-Science and Technology)
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Umbral Calculus, which is a traditional symbolic calculation method for sequence of numbers or polynomials, is provided its foundation in modern mathematical terms such as commutative algebras

and linear functional on them (This formulation is due to G.-C. Rota and collaborators). We show that the basic principle of this theory can be extended for noncommutative situations. As a byproduct, we can introduce the generalized and unified concept of noncommutative cumulants. We will also discuss some future directions of Noncommutaive Umbral Calculus.

Quantum probabilistic approach to quantum walks on spidernets

Etsuo Segawa (Department of Mathematical Informatics, University of Tokyo)
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We give an explicit expression for the spectral measure of a quantum walk (QW) on a finite/semi-infinite line. We find that 2/3 of the spectral measure on the unit circle derives from a corresponding birth and death process, and the rest of 1/3 part appears as a point mass at π. In particular, in a space homogeneous case, the spectral measure of the QW is described by the convex combination of the free-Meixner law mapped to the unit circle and a point mass at π. Furthermore, we show a relationship between the spectral measure and localization of a QW on the spidernets which has additional edges extending to the same stratum of a regular tree.

Glauber dynamics on the cycles: A quantum probabilistic view

Hyun Jae Yoo (Department of Applied Mathematics, Hankyong National University, Korea)
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We consider Glauber dynamics on finite cycles. We assume the nearest neighbor coupling constants are uniform over the edges. We will construct an interacting Fock space associated with the generator algebra, which results in a quantum decomposition of the generator. We will show that the (vacuum state) spectral measure of the generator converges, under suitable dilation and translation, to a Kesten distribution as the cycle grows to an infinite chain. As a byproduct, we will see some feature of the monotonicity of relaxation time over the coupling constants, which was conjectured by Peres and solved by Nacu for cycles.

The free Poisson limit by the free Fisher information distance

Hiroaki Yoshida (Graduate School of Humanities and Sciences, Ochanomizu University)
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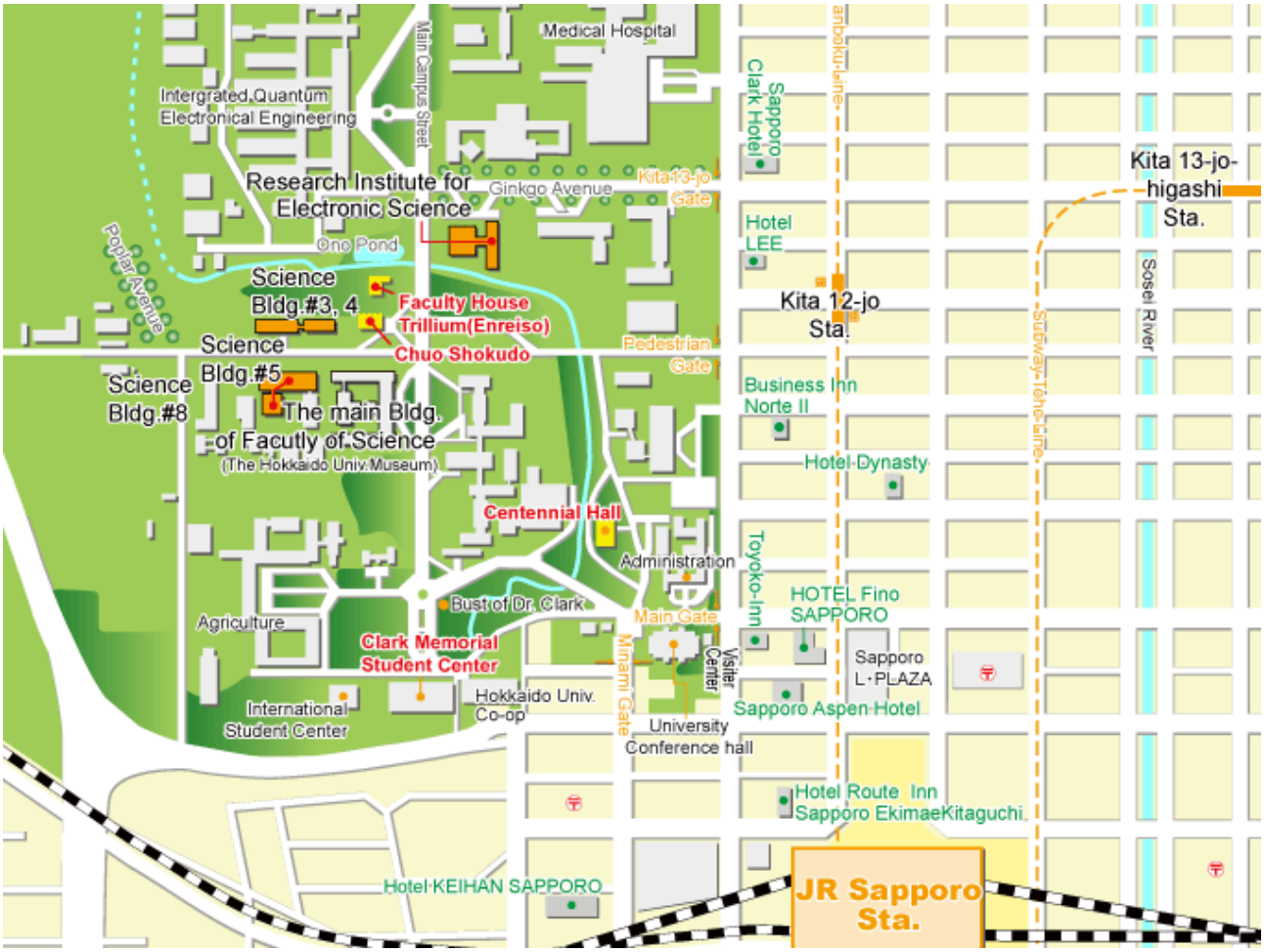
Let P and Q be (compactly supported) probability measures on $\mathbf{R}$ with $P \ll Q$. We consider a self-adjoint operator X in a W*-probability space $(\mathcal{M},\tau)$ having the distribution P with respect to the trace τ. Let $\mathcal{J}_Q(X)$ be the mismatched conjugate variable calculated by assuming X to be distributed according to Q. Then the free Fisher information distance $\Psi(P||Q)$ can be introduced by

$$\Psi(P||Q) = \tau((\mathcal{J}(X)-\mathcal{J}_Q(X))^2),$$

Where $\mathcal{J}(X)$ is the usual conjugate variable. We calculate the free Fisher information distance between the free binomial and the free Poisson laws explicitly. Applying the free logarithmic Sobolev inequality, we find that the rate of convergence of the free Poisson limit in the free relative entropy is bounded by $O(1/n^2)$, which is consistent with the classical case.



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Science Building No.3 Room 202

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http://www.math.is.tohoku.ac.jp/~obata/seminar/Sendai-WS.html