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Interview with Prof. Nobuaki Obata (CREST member)

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-- Please tell us about the Applied Mathematics Forum (AMF) you are leading.

Together with a few colleagues, I started the Applied Mathematics Forum (AMF) in September 2007 with the aim of creating a network of researchers in various fields, not limited to mathematics, but with mathematics as its core, to establish an environment where they can talk to each other at any time. This forum freely accepts anyone who wants to participate as long as they are able to support its purpose. The forum has held eight workshops in the past. The fact that more people than expected from many fields participated in the workshops has brought home to us the existence of a strong, albeit hidden, interest in mathematics. AMF is run on a volunteer basis, but it has set up a secretariat and activity base in Tohoku University's International Advanced Research and Education Organization. I want to boost the activities of the forum in conjunction with the Organization's philosophy of field integration, so that many people will find an interest in mathematics, and I ask for your cooperation in this.

-- Will you explain the relationship between your AMF and the CREST project that goes under the title "Creation of novel materials and elucidation of physical properties based on discrete geometry"?

We have recently seen a trend in the world that emphasizes applied mathematics, haven't we? As you know, an embryonic movement in support of applied mathematics has also been started in various forms here in Japan. The research field titled "Alliance for Breakthrough between Mathematics and Sciences" set up by the JST is one example of this trend, and the CREST project was one of the three themes adopted in the first year.
We could say that the research theme set up by that project came out of research seeds garnered through AMF activities. I hope that the theme will develop further under CREST just as a plant grows bigger when you feed it with fertilizer. AMF is also willing to cooperate in doing this.

-- What is your research theme?

I have been focusing on quantum probability theory for the last 15 years, a field that is not familiar in Japan. The aim of probability theory is to analyze quantities that change randomly under the laws of chance, and traditional probability theory assumes random variables are functions, that is, take numerical values. The theory clarifies the various statistical properties of random phenomena by interpreting averages of quantities such as sums and products of random variables. What I would like you to pay attention to is the fact that the multiplication of random variables is commutative because random variables are, after all, scalar functions. Mathematically speaking, random variables make up a commutative algebra. Generalizing this point, quantum probability theory allows to start with noncommutative algebras, so it is called ``noncommutative probability theory" too. The axioms of quantum probability theory can be traced back to John von Neumann, who discussed statistical problems in quantum physics in his famous book. But it has recently been developed to involve many research fields.

-- Will you explain in a little more detail how it differs from normal probability theory?

One of the typical examples where noncommutative operations appear would be with the matrix product. In short, the matrix algebra is a noncommutative algebra. By relating each matrix to a complex number (in reality, giving a positive linear function), we introduce a function that play a role of "probability." In the usual (commutative) probability theory, the mean values of XYX and X^2Y are, of course, the same. If X and Y are independent of each other, $\varphi(XYX)= \varphi(X^2Y)= \varphi(X^2)\varphi(Y)$ because of the multiplicative property of the mean values (φ denotes the mean value, also called the expectation). On the other hand, quantum probability theory does not contain the logical necessity for $\varphi(XYX)= \varphi(X^2Y)$. Instead, what makes it intriguing is that we can think of various seemingly-strange variations to the product rule. For example, the de Moivre-Laplace theorem says that the normal distribution approximates the distribution of the number of times a coin lands heads up when tossing a coin, and we can understand it as a consequence of the commutative (classical) independence in the central limit theorem. In quantum probability theory, various limit distributions appear, such as the Wigner's semicircle law or arcsine law, as consequences of the diversity of concepts of independence. In the next step, a stochastic process comes into view. As you know, a stochastic process models time-evolution of random phenomena and its randomness is understood as what is derived from accumulating small independent fluctuations over tiny time-windows. Quantum probability theory provides us a wide stage where we can play with various quantum stochastic processes based on versatile independence.

-- What do you think the extent of quantum probability theory is?

One of the topics I have become interested in lately using quantum probability theory is time-evolution of large graphs. I hope to be able to develop a new approach to real world networks or complex networks. What triggered that research was the question of what happens to the spectrum (distribution of eigenvalues of the adjacency matrix) of a Hamming graph when the length and character parameters are increased to infinity. I could find the limit without details of the spectrum of the Hamming graph by introducing a new method of ``quantum decomposition" of the adjacency matrix. This method has subsequently been refined so that it can be applied to growing regular graphs, and has been applied to many concrete examples. I want to think about the quantity for characterizing a growing network and develop the method to show what the network will look like with that quantity. As the majority of successful research efforts have been reduced down to orthogonal polynomial theory with one variable in some sense, I think that some kind of multi-variable orthogonal polynomial theory will be required in the next step, and consequently finding it will be an important task.

-- Will you explain the relationship between your research and this CREST project?

My research can be traced back to the analysis of white noise, or a more broadly speaking, to infinite-dimensional stochastic analysis. However, I started off with a stronger interest in the parts connected to quantum field theory than the stochastic process. In fact, a white noise method can be used to investigate operators in the Fock space. In the early 90s, I was working on integral representation of operators in the Fock space using white noise, and that was how I arrived at quantum probability theory. Analyzing a quantum stochastic process that develops over times is also one of the important research themes in quantum probability theory, but I have been proposing a systematic theory (quantum white noise calculus) that combines white noise analysis and quantum probability theory. I think that quantum probabilistic viewpoints will also be effective in many questions in the CREST project. I hope I will be able to contribute to deepening discussions on probability limits and scale limits in quantum probability theory, and particularly in constructing a theory of phase separation at the initial polar stage.