The analysis of singular domain perturbation problems for linear equations and systems of partial differential equations has caught the attention of several authors. In particular, a wide literature has been dedicated to the study of boundary value problems defined in domains with small holes or inclusions shrinking to points. This type of problems is of interest not only for the mathematical aspects but also in view of concrete applications to the investigation of physical models in fluid mechanics, in elasticity, and in heat conduction. For example, problems on domains with small holes or inclusions can arise in the modeling of dilute composites or of perforated elastic bodies. Investigating the effect of small perforations in the domains on the solutions of boundary value problems can help us to design objects, with improved characteristics. This is done via the so-called ‘topological optimization’ whose aim is to understand, for example, whether removing some material can improve the properties of a body. At the same time, knowing the behavior of solutions in domains with cavities can be the starting point of algorithms for the detections of holes and inclusions. However, the computational analysis of the structures consisting of components with very different lengths or dimensions often leads to numerical inaccuracy and instability. Therefore, one needs to perform a preliminary theoretical study. The most common approach to analyze problems in a domain with small holes is the one of asymptotic analysis, whose goal is to compute asymptotic expansions in terms of the size of the perforations. In this talk, we present an alternative method, i.e. the Functional Analytic Approach. Such a method aims at representing the solution or related functionals in terms of analytic maps and explicitly known functions. As a consequence one can express the solutions in terms of power series of the perturbation parameter whose coefficients can be explicitly computed. The method has revealed to be extremely versatile, with applications to several geometric settings and also to nonlinear boundary conditions.