An Epidemic Dynamics Model in a Community With Vaccinated Visitors During a Season

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1. INTRODUCTION

In our work, we focus on the effects of visitors on the epidemic dynamics of a community with ongoing vaccination during a season, if a proportion of the visitors are vaccinated. When visitors are allowed into a community during an infectious disease outbreak, there may be a presumption that there would be a resultant increase in the disease spread. However, since vaccination helps in breaking the spread of an infectious disease, what happens when a proportion of the visitors are vaccinated, becomes an interesting puzzle.

2. ASSUMPTIONS

- The disease is transmissible, but not fatal.
- The total population size of the residents is assumed to be a constant independent of time in the epidemic dynamics.
- The community accepts a constant number of visitors per unit time.
- The frequency of vaccinated visitors is given by a constant.
- Only susceptible residents are allowed to take part in the ongoing vaccination program of the community.
- Recovery from the disease, by either infected visitors or residents, has same effect as the vaccination.
- Vaccination or recovery does not offer total immunity to the disease hence, vaccinated or recovered individuals have a likelihood of being reinfected.
- A visitor must not necessarily stay throughout the season, and can also leave the community irrespective of the infectious state.

3. MODEL

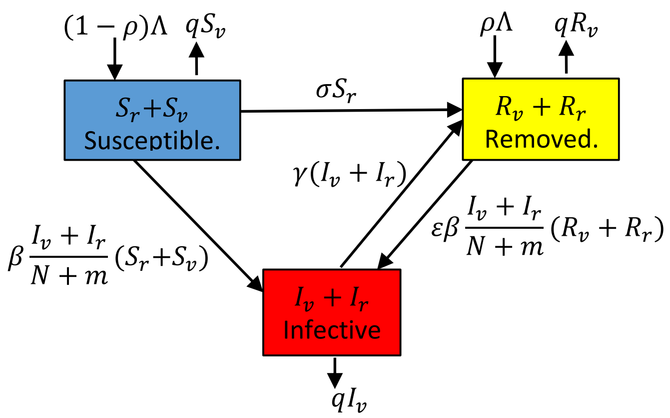

Fig. 1. The schematic diagram of the model for the epidemic dynamics of the community with visitors.

$$\text{Dynamics for the visitor:}\begin{cases}\frac{dS_v}{dt}=(1-\rho)\Lambda-\beta\frac{I_v+I_r}{N+m}S_v-qS_v;\\ \frac{dI_v}{dt}=\beta\frac{I_v+I_r}{N+m}S_v+\varepsilon\beta\frac{I_v+I_r}{N+m}R_v-\gamma I_v-qI_v\,;\\ \frac{dR_v}{dt}=\rho\Lambda+\gamma I_v-\varepsilon\beta\frac{I_v+I_r}{N+m}R_v-qR_v\,;\end{cases}$$

$$\text{Dynamics for the resident:}\begin{cases}\frac{dS_r}{dt}=-\beta\frac{I_v+I_r}{N+m}S_r-\sigma S_r;\\ \frac{dI_r}{dt}=\beta\frac{I_v+I_r}{N+m}S_r+\varepsilon\beta\frac{I_v+I_r}{N+m}R_r-\gamma I_r\,;\\ \frac{dR_r}{dt}=\sigma S_r+\gamma I_r-\varepsilon\beta\frac{I_v+I_r}{N+m}R_r\,.\end{cases}\tag{1}$$

$S_r(t)$: Susceptible residents at time t,

$S_v(t)$: Susceptible visitors at time t,

$I_r(t)$: Infective residents at time t,

$I_v(t)$: Infective visitors at time t,

$R_r(t)$:Removed residents at time t,

$R_v(t)$: Removed visitors at time t,

ρ : Frequency of vaccinated visitors,

q : Per capita emigration rate of visitors,

β : Infection rate,

γ : Recovery rate,

σ : Vaccination rate of the residents,

$\varepsilon\beta$: Reinfection coefficient for the recovered or vaccinated individuals, $0 \leq \varepsilon \leq 1$.

Λ : Net immigration rate.

$m = S_v(t) + I_v(t) + R_v(t)$: Total visitors' population density.

$N = S_r(t) + I_r(t) + R_r(t)$: Total residents' population density.

The initial condition:
$(S_v(0), I_v(0), R_v(0), S_r(0), I_r(0), R_r(0)) = ((1-\rho)m, 0, \rho m, S_{r0}, I_{r0}, 0)$.

Non-dimensional transformation of variables and parameters:
$\tau := qt$; $x_v(t) := \frac{S_v(t)}{m}$; $y_v(t) := \frac{I_v(t)}{m}$; $x_r(t) := \frac{S_r(t)}{N}$; $z(t) := \frac{I(t)}{N+m}$; $\mu := \frac{m}{N}(< 1)$; $b := \frac{\beta}{q}$; $c := \frac{\gamma}{q}$; $\omega := \frac{\sigma}{q}$.

The non-dimensionalized system:

$$\begin{aligned}
\frac{dx_v}{d\tau} &= (1-\rho) - bzx_v - x_v \\
\frac{dy_v}{d\tau} &= (1-\varepsilon)bzx_v - \varepsilon bzy_v - (c+1)y_v + \varepsilon bz\,; \\
\frac{dx_r}{d\tau} &= -bzx_r - \omega x_r; \qquad (2) \\
\frac{dz}{d\tau} &= \frac{1-\varepsilon}{1+\mu} bz(\mu x_v + x_r) - \varepsilon bz^2 - (c - \varepsilon b)z - \frac{\mu}{1+\mu} y_v.
\end{aligned}$$

4. DYNAMICS WITHOUT VISITORS

$$\begin{aligned}
\frac{dx_r}{d\tau} &= -by_r x_r - \omega x_r; \\
\frac{dy_r}{d\tau} &= (1-\varepsilon)by_r x_r - \varepsilon b{y_r}^2 - (c - \varepsilon b)y_r\,.
\end{aligned}$$

The endemic equilibrium exists if and only if $R_{00} := \frac{\varepsilon b}{c} > 1$.

Theorem 1

i. *The disease-eliminated equilibrium $E_0(0,0)$ is globally asymptotically stable if and only if $R_{00} \leq 1$.*

ii. *If and only if $R_{00} > 1$, the endemic equilibrium $E_+({x_r}^*, {y_r}^*)$ is globally asymptotically stable, while the disease-eliminated equilibrium is unstable.*

5. LOCAL STABILITY OF THE DISEASE-ELIMINATED EQUILIBRIUM FOR THE DYNAMICS WITH VISITORS

Theorem 2

If

the condition $1 - \rho < \frac{1+(1+\mu)c}{\mu(1-\varepsilon)b}\left\{\frac{1+c}{1/(1+\mu)+c} - \frac{\varepsilon b}{c}\right\}$ *(3)*

is satisfied, the disease-eliminated equilibrium is locally asymptotically stable. If the inverse inequality for (3) is satisfied, it is unstable.

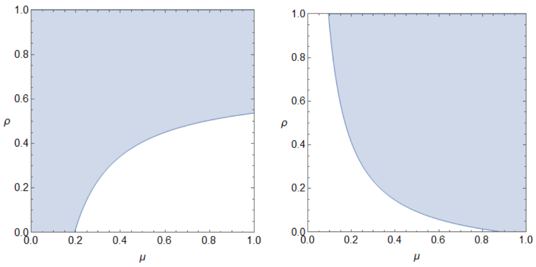

Fig. 2. Numerically drawn parameter region for ρ and μ according to (3.7). The shaded region is such that the condition (3) is satisfied. (a) $(b, \varepsilon, c) = (10, 0.55, 6.0)$, $R_{00} = 0.916667$, (b) $(b, \varepsilon, c) = (1.9, 0.55, 1.0)$, $R_{00} = 1.045$..

6. CONCLUDING REMARK

- When there is already an epidemic outbreak in the community without visitors, even with a sufficiently large proportion of vaccinated individuals, if the number of visitors allowed into the community is not above a certain threshold value, the disease might remain endemic in the community.

- When there is already an epidemic outbreak in the community without visitors, a large number of visitors with sufficiently high proportion of vaccination may suppress the outbreak.

- When the epidemic is suppressed in the community without visitors, if the number of visitors accepted into the community is sufficiently large, the epidemic might outbreak if the proportion of vaccinated visitors is not sufficiently large.

- When the epidemic is suppressed in the community without visitors, a sufficiently small number of visitors is best to keep the disease suppressed, even with the acceptance of visitors.