修士論文要旨

A mathematical model for the influence of the social insensitivity on the SIS epidemic dynamics

SIS 型感染症伝染ダイナミクスにおける社会的鈍感性の影響に関する数理モデル

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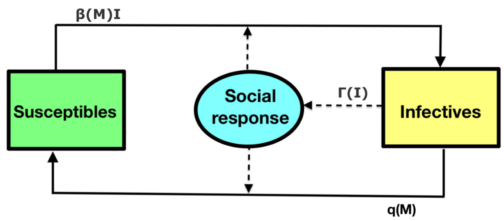
1 Introduction

The community sensitive to a disease spread generates the social response such as wearing a mask, limiting the number of contacts with others, taking medication, vaccination. Such behavioral changes may reduce the susceptibility to the disease or increase the recovery rate from it. However, the community may not respond to a transmissible disease even though such a disease is spreading in the community. A mathematical model is proposed and analyzed to consider the influence of the social insensitivity to the spread of a transmissible disease, while the infection rate and the recovery rate are affected by the social response to the disease spread.

2 Assumptions

- The disease is infatal and the disease-induced death can be negligible (for example, the common cold).
- The *social insensitivity* could be caused by the weak influence of the corresponding alert or by the unconcern to the disease spread, which are affected by the education, the culture, and the history of the community.
- The recovered individual can not get effectively long-lasting immunity and becomes susceptible again in a certain period after the recovery.
- The demographic change of the community is negligible in the time scale of considered epidemic dynamics.
- The effect of *social response* appears as the reduction of infection rate and the increase of recovery rate. For example, the social response may result in a decrease of individual contacts.
- The social response has a decay rate in time, while the existence of infectives in the community may enhance it.
- The disease spread may not enhance the social response if the number of infectives is small enough to make the people unconcern about it, that is, to cause the social insensitivity.

3 SIS modeling with social response



Susceptibles (S): individuals who are healthy and can be infected.

Infectives (I): individuals who have been infected and are able to transmit the infection.

M: the strength of the social response.

We can derive

$$\begin{aligned}\frac{dS}{dt} &= -\beta(M)IS + q(M)I\\ \frac{dI}{dt} &= \beta(M)IS - q(M)I\\ \frac{dM}{dt} &= \Gamma(I) - \mu M.\end{aligned}$$

$\beta(M)$: the infection coefficient which is a continuous and decreasing function of M with $\beta(0) = \beta_0 > 0$;

$q(M)$: the recovery rate which is a continuous and increasing function of M with $q(0) = q_0 > 0$;

$\Gamma(I)$: the social sensitivity function with $\Gamma(I) \geq 0$;

$$\Gamma(I) := \begin{cases} 0 & \text{for} \quad I \leq I_c; \\ \gamma(I - I_c) & \text{for} \quad I > I_c. \end{cases}$$

γ: the social sensitivity;

I_c: the threshold value for the number of infectives to cause the social response.

μ: the decay rate of the social response;

N: the total population size in the community which is given as $S(t) + I(t) = N > 0$ for any $t \geq 0$.

$\theta_c := I_c/N$. The basic reproduction number (基本再生産数) $\mathscr{R}_0$: the expected number of secondary infectives who is infected, in a totally susceptible community, by a single infected individual during the time span of the infection. For the generic model,

$$\mathscr{R}_0 := \frac{\beta_0 N}{q_0}.$$

4 Analytical result

Theorem

(i) If and only if $\mathscr{R}_0 \leq 1$, there is the unique disease-free equilibrium $E_0(0,0)$, which is globally asymptotically stable.

(ii) If and only if $1 < \mathscr{R}_0 \leq (1-\theta_c)^{-1}$, there are two equilibria: the disease-free equilibrium $E_0(0,0)$ and the endemic equilibrium $E_{+0}\big(N(1-\mathscr{R}_0{}^{-1}),0\big)$, of which E_0 is unstable, while E_{+0} is globally asymptotically stable.

(iii) If and only if $\mathscr{R}_0 > (1-\theta_c)^{-1}$, there are two equilibria: the disease-free equilibrium $E_0(0,0)$ and the endemic equilibrium $E_{++}(I^*, M^*)$, of which E_0 is unstable, while E_{++} is globally asymptotically stable.

Corollary *When $\mathscr{R}_0 \leq (1-\theta_c)^{-1}$, the system monotonically approaches the equilibrium.*

5 A specific model

We give specific functions for $\beta(M)$ and $q(M)$:

$$\beta(M) = \frac{\beta_0}{1+aM}; \quad q(M) = q_0 + bM.$$

We define non-dimensional transformation of variables and parameters given by

$$u = \frac{S}{N}; \quad v = \frac{I}{N}; \quad \tau = q_0 t; \quad \eta = \frac{N\gamma}{q_0};$$

$$B = \frac{b}{q_0}; \quad \delta = \frac{\mu}{q_0}; \quad \mathscr{R}_0 = \frac{\beta_0 N}{q_0}.$$

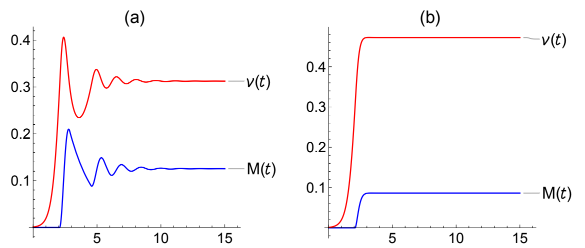
The system can be rewritten as follows:

$$\begin{aligned} \frac{du}{d\tau} &= -\frac{\mathscr{R}_0}{1+aM}uv + (1+BM)v \\ \frac{dv}{d\tau} &= \frac{\mathscr{R}_0}{1+aM}uv - (1+BM)v \\ \frac{dM}{d\tau} &= G(v) - \delta M, \end{aligned}$$

where

$$G(v) := \begin{cases} 0 & \text{for} \quad v \leq \theta_c; \\ \eta(v-\theta_c) & \text{for} \quad v > \theta_c, \end{cases}$$

and $u(t) + v(t) \equiv 1$ for any $t \geq 0$. The behavior to approach the endemic equilibrium has two different manners: damped-oscillatory or monotonic.



Numerical examples of the temporal variation with $(v(0), M(0)) = (0.001, 0)$; $\theta_c = 0.3$; $\mathscr{R}_0 = 4.0$; $a = 5.0$; $B = 5.5$; $\eta = 5.0$; (a) $\delta = 0.5$; (b) $\delta = 10.0$.

6 Conclusion

- When there is no social response, the system becomes the standard and simplest SIS model, and then shows a monotonic approach to an endemic equilibrium.
- If the community is more insensitive to the disease, the endemic size becomes larger.
- The larger decay rate of the social response increases the endemic size, and that the more sensitive social response makes the endemic size smaller.
- The more sensitive social response is more likely to cause a damped oscillation.
- The social response may play an important role to cause repetitive outbreaks in epidemic dynamics.