Hiromi Seno
Research Center for Pure and Applied Mathematics, Department of Computer and Mathematical Sciences, Graduate School of Information Sciences, Tohoku University, Sendai, Miyagi, 980-8579 Japan
e-mail: seno@math.is.tohoku.ac.jp

A Mathematical Model for Annual Variation of Incidence Size Affected by Past Epidemic Experience

Annually or seasonally characteristic fluctuation of the incidence size has been observed for a variety of infectious diseases, for example, influenza, measles, rubella, mumps, chickenpox etc. Here the *incidence size* means the *final size* of epidemic at *the epidemic season* in which the principal dynamics of disease transmission occurs. Incidence size is in general defined by the number of infected individuals, and the *relative* incidence size (or simply, incidence size) is by the fraction of infected population to the total population. Such characteristic fluctuation has been attracting many researchers in mathematical biology, bringing discussions about its driving factors. In contrast to those factors of population dynamics discussed in previous works, we focus the effect of past epidemic experience for the community on the present dynamics, including, for example, latent disease germs in the community and a change of social behavior about the prevention level which would be increased if the incidence size was large in last and/or past season(s).

To theoretically consider such effect of social factor for the annual fluctuation of incidence size about some infectious diseases, we constructed and analyzed a simple mathematical model about the annual variation of incidence size affected by incidence sizes in past seasons. We assume that the epidemic dynamics in each season is governed by the well-known Kermack–McKendrick SIR model, neglecting the temporal change of total population size within each season. We can derive the final-size equation for each season that gives $R_\infty = \lim_{t\to\infty} R(t)$ for the SIR model. Let $R_\infty(k)$ be the incidence size at the k th season, N_k be the total population size (a temporally invariant constant within the season), and $z_k = R_\infty(k)/N_k$ be the relative incidence size about the season. We hypothesize that the incidence sizes in past seasons influence the infection coefficient σ_k and the recovery rate ρ_k in the k th season, and let $\sigma_k = \sigma(\zeta_{k-1})$ and $\rho_k = \rho(\zeta_{k-1})$, where $\zeta_{k-1} := \{z_{k-1}, z_{k-2}, \dots\}$ represents the record of incidence sizes in past seasons. With this assumption to introduce the effect of past incidence sizes on the present epidemic dynamics, our modeling proposes the following mathematical model for the annual variation of incidence size affected by the social memory about the past epidemic experience:

$$-\frac{\ln(1-z_{k+1})}{z_{k+1}} = \max\left[\, \overline{\mathscr{R}}_0\,\varphi(\zeta_k),\ 1\,\right],$$

where $\overline{\mathscr{R}}_0\,\varphi(\zeta_k)$ corresponds to the effective basic reproduction number for the $k+1$ th season. Positive constant $\overline{\mathscr{R}}_0$ means the specific reproduction number which corresponds to the effective basic reproduction number at the season under condition that no outbreak of considered infectious disease occurred in the past. Function of past epidemic experience $\varphi(\zeta_k)$ introduces the effect of past incidence sizes on the epidemic dynamics in the present season.

We theoretically demonstrate that the social memory about the past epidemic experience could become a factor to cause some annual fluctuation of incidence size. Moreover we try to discuss its applicability to real epidemic data about a Japanese local community, too.

References

[1] Kermack, W.O. and McKendrick, A.G., 1927. A contribution to the mathematical theory of epidemics, *Proc. Roy. Soc*, **A115**: 700–721.

[2] Seno, H., Terada, A. and Inoue, M., 2013. A simple mathematical model for the annual variation of epidemic outbreak with prevention level affected by past incidence sizes, *RIMS Kôkyûroku*, **1853**: 134–149. (in Japanese with English abstract)