A POPULATION DYNAMICS MODEL FOR INFORMATION SPREAD UNDER THE EFFECT OF SOCIAL RESPONSE ¶

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Mark Granovetter [1, 2, 3] promoted the threshold model of social behavior in which the acceptance value of one of two distinct actions is determined by the proportion of a given population that has already accepted the action. It is about the thinking that an individual embraces an idea once a sufficient number of people has embraced the idea. This model finds application in biology, sociology, economics, information science and lots more. In this study, we develop a population dynamics model based on Granovetter's threshold hypothesis. We consider the possibility of an individual accepting and spreading some information given that a satisfactory proportion of people (threshold population) in their community is already doing the same. Given the frequency/proportion $P(t)$ of knowers of the information at a given time and the strength of social recognition effect $Q = Q(P)$ of the information, we assume that each individual is characterized by a threshold value ξ for Q, independent of time, such that

$$\begin{cases} \xi \leq Q \rightarrow \text{The individual may accept the information to transmit to others;} \\ \xi > Q \rightarrow \text{The individual ignores the information.} \end{cases}$$

The differential equation model representing this information spread behavior is given as

$$\frac{dP}{dt} = B(P(t)) \left[1 - P(t) - \int_{\Xi(P(t))} \{1 - \theta(\xi)\} f(\xi) d\xi \right],$$

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where $B(P)$ is the coefficient of information transmission to non-knowers who are willing to accept the information; $\overline{\overline{\Xi}}(P)$ is the set of threshold values for which people are not yet willing to accept the information; $\theta(\xi)$ determines the ratio of initial knowers in the subpopulation with the threshold value ξ, such that $0 \leq \theta(\xi) \leq 1$; $f(\xi)$ is the frequency distribution function (FDF) for the threshold value ξ in the population. Our analyses show that the final proportion of knowers of the information is determined by the initial proportion of knowers. We also see the existence of critical values for the initial knower size, the mean threshold value and the variance of threshold values. These critical values tend to have drastic impact on the proportion of the population that end up knowing the information.

References

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A Population Dynamics Model for Information Spread Under the Effect of Social Response

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Background

Mark Granovetter promoted the concept of threshold model of social behavior in which the acceptance value of one of two distinct actions is determined by the proportion of a given population that have already accepted the action. It is about the thinking that an individual embraces an idea once a sufficient number of people have embraced it. This model finds application in sociology, economics, information science and lots more. In this study, we develop a population dynamics model based on Granovetter's threshold hypothesis. In our study, we consider the possibility of an individual accepting and spreading some information given that a satisfactory proportion of people (threshold population) in their community are already doing the same. Our analyses show that the final proportion of knowers of the information is determined by the initial proportion of knowers. We also see the existence of critical values for the initial knower size, the mean threshold value and the variance of threshold values. These critical values tend to have drastic impact on the proportion of the population that end up knowing the information. See [1, 2, 3].

The threshold model

The general threshold distribution model for the dynamics of information spread is given as

$$\frac{dP(t)}{dt} = B(P(t))\left[1 - P(t) - \int_{\Xi(P(t))}\{1-\theta(\xi)\}f(\xi)d\xi\right]. \qquad (1)$$

The model is formulated with the following variables and parameters.

- $Q = Q(P)$: the strength of the social recognition effect, which is a function of the frequency P of knowers in the population. It is assumed to be non-decreasing in terms of P; $Q(0) = 0$, $Q \geq 0$.
- ξ: the threshold value for Q, specifying the individual independently of time.

$$\begin{cases} \xi \leq Q \rightarrow \text{The individual may accept the information to transmit to others;} \\ \xi > Q \rightarrow \text{The individual ignores the information.} \end{cases}$$

- $P(t)$: the frequency of knowers in the population at time t.

$$P(t) = \int_{-\infty}^{\infty} p(\xi, t)d\xi,$$

where $p(\xi, t)$ is the frequency distribution function (FDF) of knower's threshold value ξ in the population.

- $U(t)$: the frequency of non-knowers in the population at time t. $U(t) = \int_{-\infty}^{\infty} u(\xi, t)d\xi$, $P(t) + U(t) = 1$ independently at time t, and further $p(\xi, t) + u(\xi, t) = f(\xi)$ for any $\xi \in \mathbb{R}$ and any t.
- $\Xi(P)$: the set of the threshold value satisfying $\xi \leq Q(P)$, defined as follows: $\Xi(P) := \{\xi \mid \xi \leq Q(P)\}$, and the complementary set of $\Xi(P)$, $\overline{\Xi}(P)$, is defined by $\overline{\Xi}(P) := \{\xi \mid \xi > Q(P)\}$.
- $F(x)$: the cumulative distribution function (CDF) of the threshold value ξ in the population. $F(x) = \int_{-\infty}^{x} f(\xi)d\xi$ where $f(\xi)$ is the frequency distribution function (FDF) of the threshold value ξ in the population.
- $\mathscr{B}(\xi, P)\Delta t$:]the transition probability that the non-knower with the threshold value ξ gets the information and transits to the knower population in $[t, t+\Delta t]$ with sufficiently small Δt. $\mathscr{B}(\xi, P)$ is the coefficient of information transmission under the situation with the knower frequency P given by

$$\mathscr{B}(\xi, P) = \begin{cases} B(P), & \xi \in \Xi(P); \\ 0, & \xi \in \overline{\Xi}(P). \end{cases}$$

$B(P)$ is the coefficient of information transmission for the non-knower with the threshold value of $\Xi(P)$ with $B(0) = 0$, $B(P) > 0$ for $P \in [0, 1]$.

Model with compact support frequency distribution

In this case, the distribution of ξ is uniform with $f(\xi)$ given as

$$f(\xi) = \begin{cases} 0, & \xi < 0; \\ \frac{1}{2\bar{\xi}}, & 0 \leq \xi \leq 2\bar{\xi}; \\ 0, & \xi > 2\bar{\xi}, \end{cases} \qquad (2)$$

with mean $\bar{\xi}$.

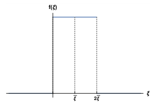

Fig. 1: Graph of the frequency distribution function $f(\xi)$ against the threshold value ξ of the social recognition effect given by (2).

The model is expressed as

$$\frac{dP(t)}{dt} = \begin{cases} B(P(t))\left[\theta_0 - \left(1 - \frac{\alpha}{2\bar{\xi}}(1-\theta_0)\right)P(t)\right], & \alpha P(t) \leq 2\bar{\xi}; \\ B(P(t))[1 - P(t)], & \alpha P(t) > 2\bar{\xi}. \end{cases} \qquad (3)$$

with $\theta(\xi) = \theta_0$ and $Q(P(t)) := \alpha P(t)$.

Model with everywhere positive distribution

We have

$$\frac{dP(t)}{dt} = B(P(t))G(P(t)) \qquad (4)$$

with initial condition $P(0) = P_0 = \theta_0$ and

$$G(P) := 1 - P - (1-\theta_0)\int_{Q(P)}^{\infty} f(\xi)d\xi. \qquad (5)$$

$G(P)$ is continuous in terms of P and $f(\xi)$ is positive for every real threshold value ξ. At the equilibrium state $P = P^*$ where $dP(t)/dt = 0$, we have $G(P^*) = 0$ since $B(P) > 0$. With the specific distribution

$$f(\xi) = \frac{1}{\sigma\sqrt{2}}e^{-\sqrt{2}\frac{|\xi-\bar{\xi}|}{\sigma}} = \begin{cases} \frac{1}{\sigma\sqrt{2}}e^{\sqrt{2}\frac{(\xi-\bar{\xi})}{\sigma}}, & \xi < \bar{\xi}; \\ \frac{1}{\sigma\sqrt{2}}e^{-\sqrt{2}\frac{(\xi-\bar{\xi})}{\sigma}}, & \xi \geq \bar{\xi}, \end{cases} \qquad (6)$$

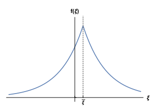

Fig. 2: Graph of the frequency distribution function $f(\xi)$ against the threshold value ξ of the social recognition effect given by (6).

we now have

$$G(P) = \begin{cases} \mathcal{G}_1(P) := \theta_0 - P + \frac{1}{2}(1-\theta_0)e^{\sqrt{2}\frac{\alpha}{\sigma}\left(P-\frac{\bar{\xi}}{\alpha}\right)}, & P < \bar{\xi}/\alpha; \\ \mathcal{G}_2(P) := 1 - P - \frac{1}{2}(1-\theta_0)e^{-\sqrt{2}\frac{\alpha}{\sigma}\left(P-\frac{\bar{\xi}}{\alpha}\right)}, & P \geq \bar{\xi}/\alpha. \end{cases} \qquad (7)$$

Equilibrium value of P

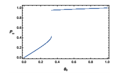

Fig. 3: Numerically obtained convergence for P with parameter θ_0 with $\bar{\xi}/\alpha = 0.55$, $\sigma/\alpha = \sqrt{2}/10$.

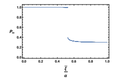

Fig. 4: Numerically obtained convergence for P with parameter $\bar{\xi}/\alpha$ with $\theta_0 = 0.3$, $\sigma/\alpha = \sqrt{2}/10$.

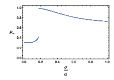

Fig. 5: Numerically obtained convergence for P with parameter σ/α with $\theta_0 = 0.3$, $\bar{\xi}/\alpha = 0.55$.

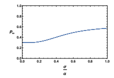

Fig. 6: Numerically obtained convergence for P with parameter σ/α with $\theta_0 = 0.3$, $\bar{\xi}/\alpha = 0.75$.

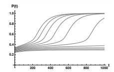

Fig. 7: Temporal variation of $P(t)$ with varying initial values $P(0) = \theta_0$.

Concluding remarks

The final proportion/frequency of knowers is largely dependent on the initial proportion of knowers. Critical conditions relating to heterogeneity of individuals in a population determine the consequence of information spread. For proper information dissemination, it is necessary to take into account the social situation of a population and the nature of information under consideration.

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