## **A Mathematical Consideration**

## for the Effect of Regional Lockdown on Endemic Size

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Lockdown is one of the effective methods to prevent the further spread of an epidemic, though it may bring about economic and social difficulty in the community. To balance the epidemic control and social activities, the policymaker needs to choose a better policy to take account of a balance of them. In this work, we consider the following mathematical model of an epidemic dynamics in a community composed of the peripheral area (area 1) and central area (area 2) which respectively have different qualities about the medical treatment for the disease.

$$\begin{split} \frac{dS_1}{dt} &= -\beta_1 I_1 S_1 - \alpha_1 \beta_2 I_2 S_1 + \theta_1 H_{11} + \theta_2 H_{21}; & \frac{dS_2}{dt} = -\beta_2 I_2 S_2 - \alpha_2 \beta_1 I_1 S_2 + \theta_2 H_{22}; \\ \frac{dI_1}{dt} &= \beta_1 I_1 S_1 + \alpha_1 \beta_2 I_2 S_1 - \gamma_1 I_1; & \frac{dI_2}{dt} = \beta_2 I_2 S_2 + \alpha_2 \beta_1 I_1 S_2 - \gamma_2 I_2; \\ \frac{dH_{11}}{dt} &= (1-p) \gamma_1 I_1 - \theta_1 H_{11}; & \frac{dH_{22}}{dt} = \gamma_2 I_2 - \theta_2 H_{22}, \\ \frac{dH_{21}}{dt} &= p \gamma_1 I_1 - \theta_2 H_{21}; \end{split}$$

where  $S_i$  and  $I_i$  are susceptible and infective population densities in area *i* respectively.  $H_{ij}$  denotes the population density of infected individuals who belong to area *j* and are isolated under the medical treatment in area *i*. Coefficient  $\beta_i$  is of the infection at area *i* for the individual belonging to area *i*, and  $\alpha_i \beta_i$  is of the infection at area j for the individual belonging to area i during the temporary visit to area j, which is smaller than  $\beta_i$  ( $0 < \alpha_i < 1$ ). In this sense, the parameter  $\alpha_i$  can be regarded as an index of the mobility of susceptible individuals belonging to area i. Parameter  $\gamma_i$  is the detection/isolation rate of the infective individual belonging to area i, and  $\theta_i$  is the discharge rate from the isolation under the medical treatment at area i. The proportion p of infectives belonging to area 1 get the medical treatment in the central area ( $0 \le p \le 1$ ). We introduce different restrictions on the mobility of individuals and define four types of lockdown: complete lockdown ( $\alpha_1 = \alpha_2 = p = 0$ ), strong lockdown ( $\alpha_1 = \alpha_2 = 0$ ), weak lockdown type 1 ( $\alpha_1 = 0$ ), and weak lockdown type 2 ( $\alpha_2 = 0$ ) 0). Then we compare the efficiencies of those lockdowns according to the endemic size, that is, the number of infected individuals at the endemic equilibrium. The endemic size is defined by  $\Psi^*$ :=  $(N_1 + N_2 - S_1^* - S_2^*)/(N_1 + N_2)$  where  $N_i$  is the population size in area *i*. The mathematical analysis shows the existence and stability of possible equilibria under each of lockdowns for our model. In this presentation, we are going to show the result on which type of lockdown could be regarded as better according to the suppression of the endemic size.