

How many preys could coexist with a shared predator in the Lotka-Volterra system?

ロトカ・ボルテッラ系において捕食者 1 種はどのくらいの数の餌種と共存できるか？

Hiromi Seno^{1,*}, Victor Schneider¹ and Toshihiko Kimura²

¹ 東北大学大学院情報科学研究科, ² 広島大学理学部数学科

*seno@math.is.tohoku.ac.jp

The interspecific interaction in a food web is made up of *direct* and the *indirect* effects. Direct effect includes competition, predation and symbiosis. Indirect effect is defined as an effect on a species from another which has no direct interaction with it. The indirect effect between two species could occur through interactions with the other species in the food web. *Apparent competition* is defined by Holt (1977, 1984) as a negative indirect effect between two prey species which have a shared predator and have no direct interaction between them. In a system of one predator and its two prey species, one prey population plays a role to increase the predator population, so that the other prey population can be regarded as *indirectly* affected by the former prey population even if no direct interaction exists between them. There have been lots of previous ecological works related to apparent competition, in which the effect of predation on the diversity of competing prey species was mainly discussed. However, the “apparent competition” effect defined above has been accepted and it is used today for the theoretical discussions in a variety of contexts which transcend ecology (Holt and Bonsall, 2017), including pest control, immune dynamics and epidemics. In nature, the members of a food web are always subjected to change on a long time scale following species extinctions and invasions. Morris *et al.* (2004) successfully demonstrated the long-term apparent competition in natural communities of herbivorous insects, and gave a suggestion that interactions mediated by shared natural enemies may be a significant factor in structuring natural communities. In lots of theoretical researches about the effect of the species deletion or introduction on the community structure, community assembly models or “global models” has been constructed, analyzed and investigated mainly to discuss the stability of structure.

In this work, we analyze the Lotka-Volterra n preys-1 predator system in which prey species have no direct interaction among them (Seno *et al.*, 2020). Prey species have only indirect interactions, that is, apparent competition via the shared predator. We revisit the system analyzed in Holt (1977) and we find the necessary and sufficient condition to determine which equilibrium states becomes globally asymptotically stable, because the system necessarily has a globally asymptotically stable unique equilibrium state. Then we shall focus on the transition of the equilibrium state due to the deletion or the introduction of a prey species into the system. With this, we discuss in a systematic manner how many prey species a generalist predator could coexist with and how the apparent competition works to balance the equilibrium state.

References

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