A mathematical model for the dynamics of information spread under the effect of social response

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Mark Granovetter promoted the concept of threshold model of social behavior in which the acceptance value of one of two distinct actions is determined by the proportion of a given population that has already accepted the action. It is about the thinking that an individual embraces an idea once a sufficient number of people has embraced the idea. This model finds application in sociology, economics, information science and lots more.

In this study, we develop a population dynamics model based on Granovetter's threshold hypothesis. We conssider the possibility of an individual accepting and spreading some information given that a satisfactory proportion of people (threshold population) in their community is already doing the same. Given the frequency/proportion $P(t)$ of knowers of the information at a given time and the strength of social recognition effect $Q = Q(P)$ of the information, we assume that each individual is characterized by a threshold value ξ for Q, independent of time, such that

$$\begin{cases} \xi \leq Q \rightarrow \text{The individual may accept the information to transmit to others;} \\ \xi > Q \rightarrow \text{The individual ignores the information.} \end{cases}$$

The differential equation model representing this information spread behavior is given as

$$\frac{dP(t)}{dt} = B(P(t)) \left[1 - P(t) - \int_{\overline{\Xi}(P(t))} \{1 - \theta(\xi)\} f(\xi) d\xi \right],$$

where $B(P)$ is the coefficient of information transmission to non-knowers who are willing to accept the information; $\overline{\Xi}(P)$ is the set of threshold values for which people are not yet willing to accept the information; $\theta(\xi)$ determines the ratio of initial knowers in the subpopulation with the threshold value ξ, such that $0 \leq \theta(\xi) \leq 1$; $f(\xi)$ is the frequency distribution function (FDF) for the threshold value ξ in the population.

Our analyses show that the final proportion of knowers of the information is determined by the initial proportion of knowers. We also see the existence of critical values for the *initial knower size*, the *mean threshold value* and the *variance* of threshold values. These critical values tend to have drastic impact on the proportion of the population that end up knowing the information.