A mathematical model for the influence of the social insensitivity on the SIS epidemic dynamics

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When a transmissible disease invades, the community may respond to the disease in such a way of wearing masks to reduce the infection probability or getting the vaccine to prevent the serious symptom and the disease transmission. However, the community may not necessarily show such a social response to a transmissible disease, being insensitive to the disease spread. In this work, we propose a specific Susceptible-Infective-Susceptible (SIS) model, taking account of the effect of such social response. We assume the followings:

- The disease is infatal;
- The recovered individual can not get effectively long-lasting immunity but become susceptibles again insufficiently short period after the recovery;
- The demographic change of the population size is negligible in the time scale of the considered epidemic dynamics;
- The social response has a decay rate in time, the number of infectives in the community tends to enhance its effect, which reduces the infection rate and increases the recovery rate.

The model is given by the following system of ordinary differential equations:

$$\begin{aligned} \frac{dS}{dt} &= -\beta(M)SI + q(M)I, \\ \frac{dI}{dt} &= \beta(M)SI - q(M)I, \\ \frac{dM}{dt} &= \Gamma(I) - \mu M, \end{aligned} \tag{1}$$

where $M = M(t)$ is the strength of the social response at time t, which has the natural decay rate μ. $\Gamma(I)$ is called the social sensitivity function of the number of infectives I, which could reflect the cultural characteristics, the social environment, and the educational circumstance. We introduce a critical value I_c for the number of infectives below which the community is insensitive to the disease, and the function Γ of I is given by

$$\Gamma(I) := \begin{cases} 0 \quad \text{for} \quad I \le I_c, \\ \gamma(I - I_c) \quad \text{for} \quad I > I_c. \end{cases} \tag{2}$$

In this work, we are going to discuss the effect of the social insensitivity on the epidemic dynamics, especially focusing on the nature of the endemic state, since the system (1) necessarily approaches an endemic state.