病原体個体群動態を含む数理モデリングによる SIR モデル

An SIR modeling with the pathogen population dynamics of disease transmission

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In this work, we consider an SIR model of infectious disease transmission through the outside environment of host individual, for example, like an aerial infection. The pathogen population around the host population is assumed to be reproduced by the infective hosts themselves. Furtherthe dispersal of pathogen individuals outside of the host is assumed to be passive, and sufficiently fast outside of the host.

Susceptible individual becomes infective with the successful disease transmission, and then comes to have an infectivity. Now we define the infectivity as the host state in which the pathogen in the host reproduces itself and emigrates outside from the host body. Although the immune system of host may decrease and eliminate the pathogen within the host body, we assume that the host necessarily releases pathogens reproduced in the host body.

With these assumptions, we consider the following SIR model:

$$\begin{aligned}
\frac{dS}{dt} &= -\sigma(\kappa_{\mathrm{S}}P)S \\
\frac{dI}{dt} &= \sigma(\kappa_{\mathrm{S}}P)S - \rho(\kappa_{\mathrm{I}}P)I \\
\frac{dR}{dt} &= \rho(\kappa_{\mathrm{I}}P)I \\
\epsilon\frac{dP}{dt} &= mS - \kappa_{\mathrm{S}}SP - \kappa_{\mathrm{I}}IP - \kappa_{\mathrm{R}}RP - qP,
\end{aligned}$$

where $S = S(t)$, $I = I(t)$ and $R = R(t)$ are respectively the susceptible, the infective and the removed host population at time t with $S + I + R = N$ at any time t. N is the population size of host, which is now assumed to be a constant independent of time. $P = P(t)$ is the mean density of dispersing pathogen population over the host population at time t. Parameters m, κ_{S}, κ_{I}, κ_{R}, and q are positive constants.

In the above model,we give the probability of the host state transition from 'susceptible' to 'infective' with the immigration and the settlement of dispersing pathogen by $\sigma(\kappa_{\mathrm{S}}P)\Delta t$ for sufficiently short time interval $[t, t+\Delta t]$ in the time scale of the host state transition. Similarly, the probability of host state transition from 'infective' to 'removed' due to the recovery with immunity or the death is given by $\rho(\kappa_{\mathrm{I}}P)\Delta t$ for sufficiently short time interval $[t, t+\Delta t]$ in the time scale of the host state transition.

The positive parameter ϵ has the meaning that the representative time scale for the reproduction and dispersal of pathogen population is ϵ when that for the host state transition is unity. Since the reproduction and dispersal of pathogen population is now assumed to be the "fast" process, we put $\epsilon \ll 1$. Applying the quasi stationary state approximation for the above model, in the presentation, we show a flexibility of the above model to discuss the characteristics of disease transmission with some different natures, including the basic reproduction number R_0.