A Population Dynamics Model for the Information Spread under the Effect of Social Response

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We construct and analyze a mathematically reasonable and simplest discrete time one dimensional population dynamics model based on Mark Granovetter's idea for the spread of a matter (rumor, innovation, etc.) in a population. Individual threshold values with respect to the decision making on the acceptance of a spreading matter are distributed throughout the population. We give the mathematical results on how the equilibrium acceptor frequency depends on the nature of threshold distribution in the population.

Granovetter's threshold model

The essential idea for the dichotomous decision-making by an individual on a spreading matter in the population, that is, about whether he/she accepts it or not:

- The individual has a criterion only by which the decision is made.
- Each individual's criterion for the decision-making may different from some others'.
- The criterion is more likely to be satisfied as the accepter frequency gets larger in the community.

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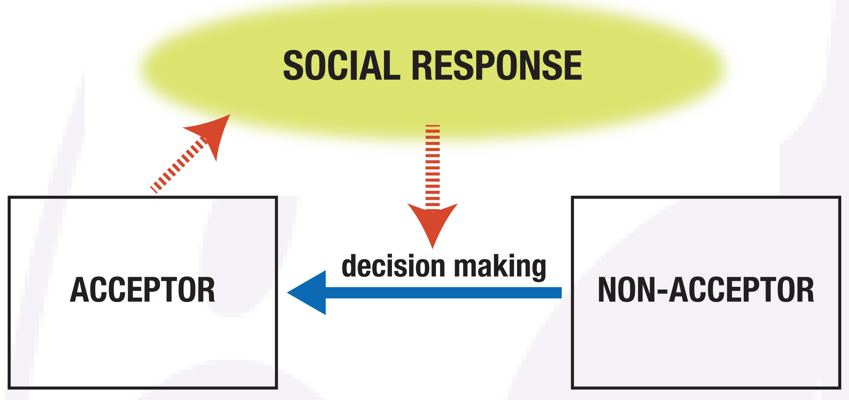
Assumptions



1. The criterion of decision-making by each individual is given by the threshold for the strength of social effect.
2. The strength of social effect is proportional to the accepter frequency in the community.
3. The decision to accept the spreading matter may be done only when the strength of social effect is beyond the threshold.
4. The threshold of one individual may different from that of the other. (threshold $\in$ individuality)
5. Every decision-making is independent of the past disregard/denial about the spreading matter.
6. The accepter never discards the accepted matter.

Decision-making rule to accept the spreading matter

$$\begin{cases} \xi \le \alpha P \Longrightarrow \text{The matter may be accepted.} \\ \xi > \alpha P \Longrightarrow \text{The matter is not accepted with the denial/disregard.} \end{cases}$$

ξ : the threshold value of the individual.

P : the acceptor frequency in the community.

αP : the strength of social consciousness according to the spreading matter, now assumed to be proportional to the acceptor frequency.

Threshold distribution

$$F(x) = \text{Prob}(\xi \le x) = \int_{-\infty}^{x} f(\xi)\, d\xi$$

with

$$f(\xi) = \begin{cases} 0 & \text{for } \xi \in (-\infty,\ \xi_{\inf}]; \\ f_+(\xi) & \text{for } \xi \in (\xi_{\inf},\ \xi_{\sup}) \subset (0, \alpha); \\ 0 & \text{for } \xi \in [\xi_{\sup},\ \infty), \end{cases}$$

$$\int_{\xi_{\inf}}^{\xi_{\sup}} f_+(\xi)\, d\xi = 1.$$

Initial acceptor frequency

$$P_0 = \int_{-\infty}^{\infty} \varphi_0 f(\xi)\, d\xi = \varphi_0 \int_0^{\alpha} f(\xi)\, d\xi = \varphi_0$$

as randomly chosen initial acceptors with $0 < \varphi_0 \le 1$.

Recurrence relation for the temporal sequence of acceptor frequency

$$P_{t+1} = \{1 - \gamma B(P_t)\}P_t + \gamma B(P_t)\Big\{F(\alpha P_t) + \int_{\alpha P_t}^{\alpha} \varphi_0(\xi) f(\xi)\, d\xi\Big\}$$
$$= \Big[1 + \gamma b\{\varphi_0 - P_t + (1-\varphi_0)F(\alpha P_t)\}\Big]P_t = \begin{cases} \big[1 + \gamma b \mathscr{G}(P;\varphi_0)\big]P_t & \text{for } P_t \in [\varphi_0, \theta_{\sup}); \\ \big[1 + \gamma b(1 - P_t)\big]P_t & \text{for } P_t \in [\theta_{\sup}, 1], \end{cases}$$

with $\mathscr{G}(P;\varphi_0) := \varphi_0 - P + (1-\varphi_0)\int_{\xi_{\inf}}^{\alpha P} f_+(\xi)\, d\xi.$

P_t : Acceptor frequency at time t.

αP_t : Strength of social effect at time t.

$B(P) = bP$: Probability to get the chance for the decision-making under the accepter frequency P. $(0 < b \le 1)$

γ : Probability to make the decision to accept the spreading matter per chance.

Theorem

The sequence $\{P_t\}$ monotonically increases and converges to a value $P^* \in [P_0, 1]$ for any $P_0 \in (0,1)$ as $t \to \infty$.

Lemma

If $P_0 = \varphi_0 \le \theta_{\inf} := \xi_{\inf}/\alpha$, the acceptor frequency remains the initial frequency P_0 with no increase at any time step. Otherwise, it temporally increases.

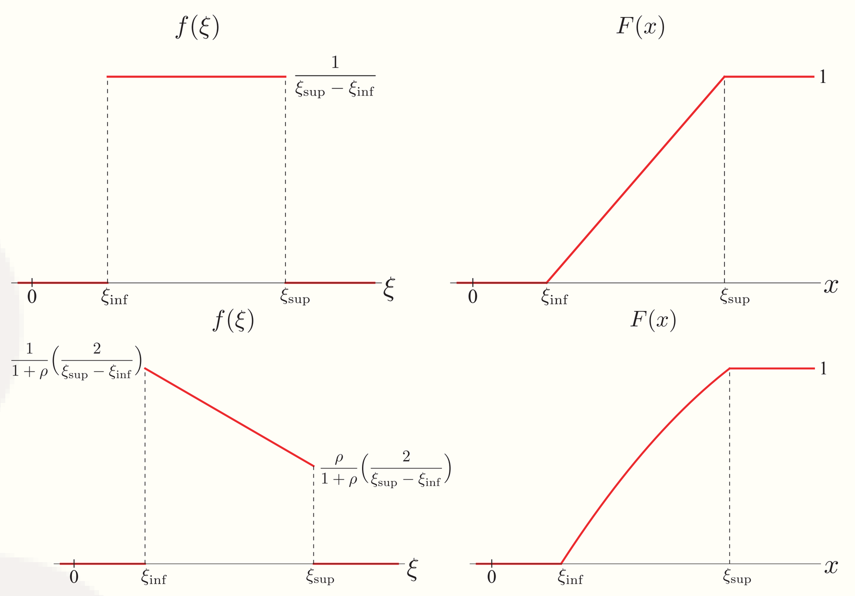
Lemma

If $P_0 = \varphi_0 \ge \theta_{\sup} := \xi_{\sup}/\alpha$, the acceptor frequency monotonically increases toward 1.

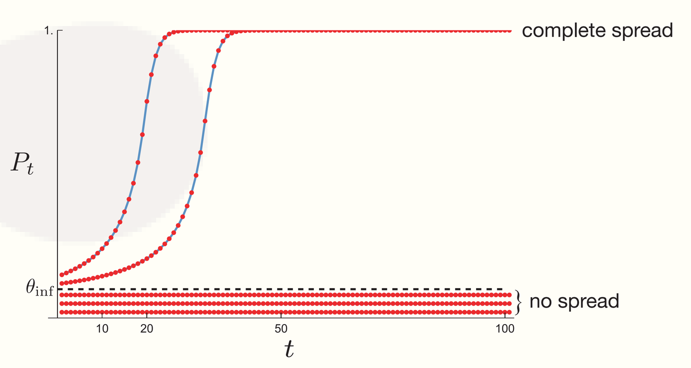
Lemma

If the equation $\mathscr{G}(P;\varphi_0) = 0$ in terms of P has no root in $(\varphi_0, \theta_{\sup}) \subset (\theta_{\inf}, \theta_{\sup})$, then $P_t \to 1$ as $t \to \infty$ for $P_0 = \varphi_0 \in (\theta_{\inf}, \theta_{\sup})$.

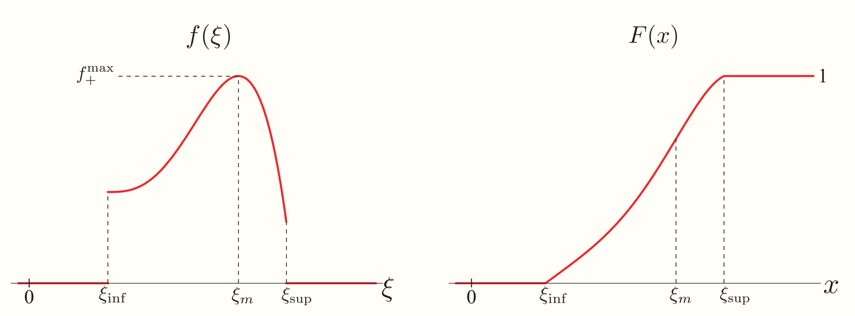
Uniform or monotonically decreasing threshold distribution



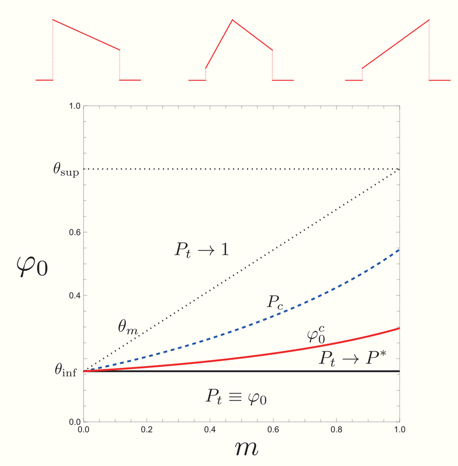
For the uniform or monotonically decreasing threshold distribution, the acceptor frequency P_t monotonically approaches 1 as time passes for the initial acceptor frequency such that $P_0 = \varphi_0 > \theta_{\inf}$, while it remains the initial frequency, $P_t \equiv \varphi_0$, for $\varphi_0 \le \theta_{\inf}$.



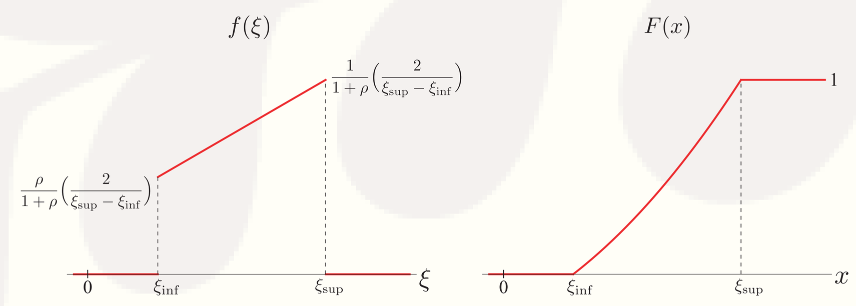
Unimodal threshold distribution



The behavioral characteristics of the acceptor frequency is qualitatively same as for the monotonically increasing distribution.



Monotonically increasing threshold distribution



For the monotonically increasing threshold distribution, the acceptor frequency P_t has the following behavior as time passes, depending on the nature of threshold distribution and the initial acceptor frequency φ_0: If

$$\alpha f_+^{\inf} := \lim_{\xi \to \xi_{\inf}+0} \alpha f_+(\xi) < \frac{1}{1-\theta_{\inf}}, \qquad (\star)$$

then

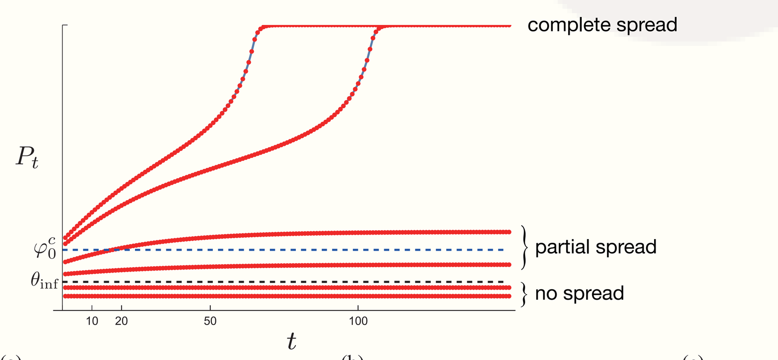
$$\begin{cases} P_t \equiv P_0 = \varphi_0 & \text{for } \varphi_0 \in [0, \theta_{\inf}]; \\ P_t \to P^* & \text{for } \varphi_0 \in (\theta_{\inf}, \varphi_0^c] \subset (\theta_{\inf}, \theta_{\sup}); \\ P_t \to 1 & \text{for } \varphi_0 \in (\varphi_0^c, 1], \end{cases}$$

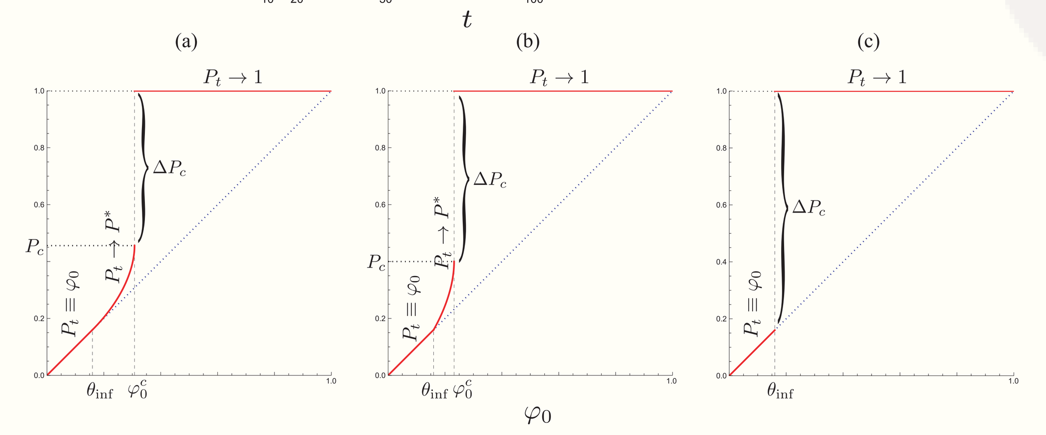
where P^* is uniquely determined as the smallest root $P = P^* \in (\varphi_0, \theta_{\sup})$ of the equation $\mathscr{G}(P;\varphi_0) = 0$ and $\varphi_0^c = 1 - 1/\{\alpha f_+(\alpha P_c)\}$ with the unique root $P = P_c \in (\theta_{\inf}, \theta_{\sup})$ of the equation

$$Q(P) := 1 - (1-P)\alpha f_+(\alpha P) - \int_{\xi_{\inf}}^{\alpha P} f_+(\xi)\, d\xi = 0.$$

Unless the condition $(\star)$ is satisfied, then

$$\begin{cases} P_t \equiv P_0 = \varphi_0 & \text{for } \varphi_0 \in [0, \theta_{\inf}]; \\ P_t \to 1 & \text{for } \varphi_0 \in (\theta_{\inf}, 1]. \end{cases}$$





(a) $\rho = 0.0$ ($P_c = 0.456$; $\varphi_0^c = 0.308$); (b) $\rho = 0.2$ ($P_c = 0.400$; $\varphi_0^c = 0.232$); (c) $\rho = 0.7$, and commonly $\theta_{\inf} = 0.16$; $\theta_{\sup} = 0.8$; $\rho_c = 0.615$.

- There exists a critical value for the initial acceptor frequency, beyond which the spread of a matter becomes highly successful.
- Successful spread of a matter is more likely to occur for a community such that the members have the decision-making threshold relatively biased to the smaller value.
- This model could provide a basic mathematical structure for modified models about a variety of theoretical problems on a spreading matter in a community.

References

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