Social response could cause recurring epidemic outbreaks: A mathematical model

Ying XIE [1,*], Hiromi SENO [1]

[1]Graduate School of Information Sciences, Tohoku University, Sendai, Japan

Introduction

When a transmissible disease invades, the community may respond to the disease in such a way as wearing masks to reduce the infection risk or by getting the vaccine to prevent the serious symptoms and the disease transmission. In contrast, the community may be insensitive to the disease. We consider a mathematical model based on the Susceptible-Infective-Susceptible (SIS) model, taking account of the effect of social response on the epidemic dynamics of a transmissible disease. Our results show the possible contribution of the social sensitivity and insensitivity to the occurrence of an oscillatory variation in the epidemic dynamics, which could be observed as recurring outbreaks.

Assumptions

- The spreading disease is non-fatal, and the disease-induced death can be negligible (for example, the common cold).
- The recovered individual cannot get the long-lasting effective immunity and becomes susceptible again in a sufficiently short period after the recovery.
- The demographic change about the community is negligible in the time scale of considered epidemic dynamics.
- The stronger social response makes the infection rate smaller, for example, with a decrease of individual contact rate.
- The social response follows a natural decay, while the fact of disease spread in the community tends to arouse the response.
- The disease spread may not cause the social response unless the number of infectives becomes enough to concern the people about it. Such a situation defines the social insensitivity. It may depend on the educational or cultural backgrounds of the community members.

Modeling and generic model

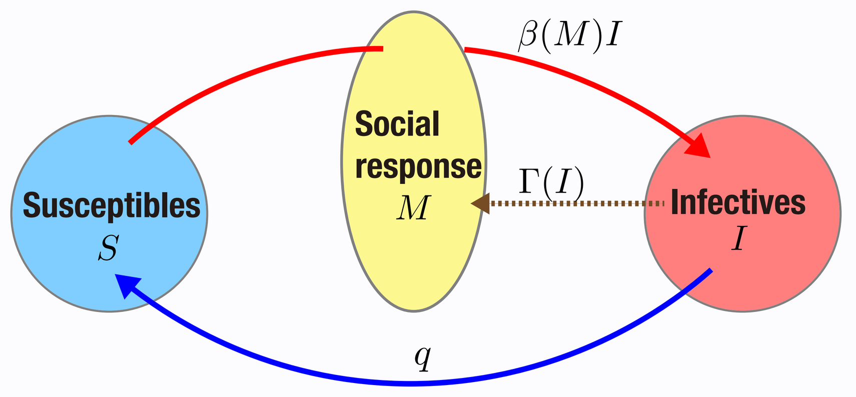

Figure 1. The epidemic state transition and the social response about our model.

$$\frac{dS}{dt} = -\beta(M)IS + qI;\quad \frac{dI}{dt} = \beta(M)IS - qI;\quad \frac{dM}{dt} = \Gamma(I) - \mu M,$$

$$\Gamma(I) := \begin{cases} 0 & \text{for } I \le I_c; \\ \gamma(I - I_c) & \text{for } I > I_c. \end{cases}$$

- $S(t)$: the susceptible population density in the community at time t;
- $I(t)$: the infective population densitiey in the community at time t;
- $M(t)$: the strength of the social response at time t;
- $\beta(M)$: the disease transmission coefficient with $\beta(0) = \beta_0 > 0$ and $\beta'(M) = d\beta(M)/dM > 0$;
- q: the recovery rate;
- $\Gamma(I)$: the social sensitivity function with $\Gamma(I) \ge 0$.
- μ: the decay rate of the social response;
- γ: the social sensitivity coefficient;
- I_c: the threshold infective density to raise the social response;
- N: the total population size $N = S(t) + I(t)$.

Non-dimensionalization

$$u := \frac{S}{N};\ v := \frac{I}{N};\ \tau := qt;\ \eta := \frac{\gamma N}{q};\ \theta_c := \frac{I_c}{N};\ \delta := \frac{\mu}{q};$$

$\mathscr{R}_0 := \frac{\beta_0 N}{q}$ (basic reproduction number).

$$\frac{dv}{d\tau} = \frac{\beta(M)}{\beta_0}\mathscr{R}_0 v(1-v) - v;\quad \frac{dM}{d\tau} = G(v) - \delta M,$$

$$G(v) := \begin{cases} 0 & \text{for } v \le \theta_c; \\ \eta(v - \theta_c) & \text{for } v > \theta_c \end{cases}$$

with the initial condition that $v(0) > 0$ and $M(0) = 0$.

Model with no social response ($M \equiv 0$)

$$\frac{dv}{d\tau} = \mathscr{R}_0 v(1-v) - v,$$

$$v(\tau) = \begin{cases} \dfrac{v_0(1-1/\mathscr{R}_0)}{v_0 + \{(1-1/\mathscr{R}_0) - v_0\}e^{-\tau/(\mathscr{R}_0 - 1)}} & \text{for } \mathscr{R}_0 \ne 1; \\ \dfrac{1}{\tau + 1/v_0} & \text{for } \mathscr{R}_0 = 1. \end{cases}$$

Model without social insensitivity ($\theta_c = 0$)

$$\frac{dv}{d\tau} = \frac{\beta(M)}{\beta_0}\mathscr{R}_0 v(1-v) - v;\quad \frac{dM}{d\tau} = \eta v - \delta M.$$

Theorem 1. For the model without social insensitivity,

(i) *if and only if $\mathscr{R}_0 \le 1$, the unique equilibrium $E_0(0,0)$ is globally asymptotically stable;*

(ii) *if and only if $\mathscr{R}_0 > 1$, there are two equilibria $E_0(0,0)$ and $E_+(v^*, M^*)$, where E_0 is unstable and E_+ is globally asymptotically stable with v^* and M^* uniquely determined by*

$$v^* = 1 - \frac{1}{\mathscr{R}_0}\frac{\beta_0}{\beta(M^*)};\quad M^* = \frac{\eta}{\delta}v^*.$$

where $0 < v^* < 1 - 1/\mathscr{R}_0$.

Model with social insensitivity ($\theta_c > 0$)

Theorem 2. For the model with social insensitivity,

(i) *if and only if $\mathscr{R}_0 \le 1$, the unique equilibrium E_0 is globally asymptotically stable;*

(ii) *if and only if $1 < \mathscr{R}_0 \le (1-\theta_c)^{-1}$, there are two equilibria E_0 and E_{+0}, where E_0 is unstable and $E_{+0}(1 - 1/\mathscr{R}_0, 0)$ is globally asymptotically stable;*

(iii) *if and only if $\mathscr{R}_0 > (1-\theta_c)^{-1}$, there are two equilibria E_0 and $E_{++}(v^*, M^*)$, where E_0 is unstable and E_{++} is globally asymptotically stable with v^* and M^* uniquely determined by*

$$v^* = 1 - \frac{1}{\mathscr{R}_0}\frac{\beta_0}{\beta(M^*)};\quad M^* = \frac{\eta}{\delta}(v^* - \theta_c),$$

where $\theta_c < v^* < 1 - 1/\mathscr{R}_0$.

Corollary 1. The system approaches

(i) *the equilibrium E_0 in a monotonic manner when $\mathscr{R}_0 \le 1$;*

(ii) *the equilibrium E_{+0} in a monotonic manner when $1 < \mathscr{R}_0 \le (1-\theta_c)^{-1}$;*

(iii) *the equilibrium E_{++} in the following manner when $\mathscr{R}_0 > (1-\theta_c)^{-1}$:*

$$\begin{cases} \text{a monotonic manner if } \Delta \ge 0; \\ \text{an oscillatory manner if } \Delta < 0, \end{cases}$$

where

$$\Delta := \left\{\frac{\beta(M^*)}{\beta_0}\mathscr{R}_0 v^* + \delta\right\}^2 + 4\eta v^* \frac{\beta'(M^*)}{\beta(M^*)}.$$

A specific model

$$\beta(M) = \frac{\beta_0}{1 + aM}$$

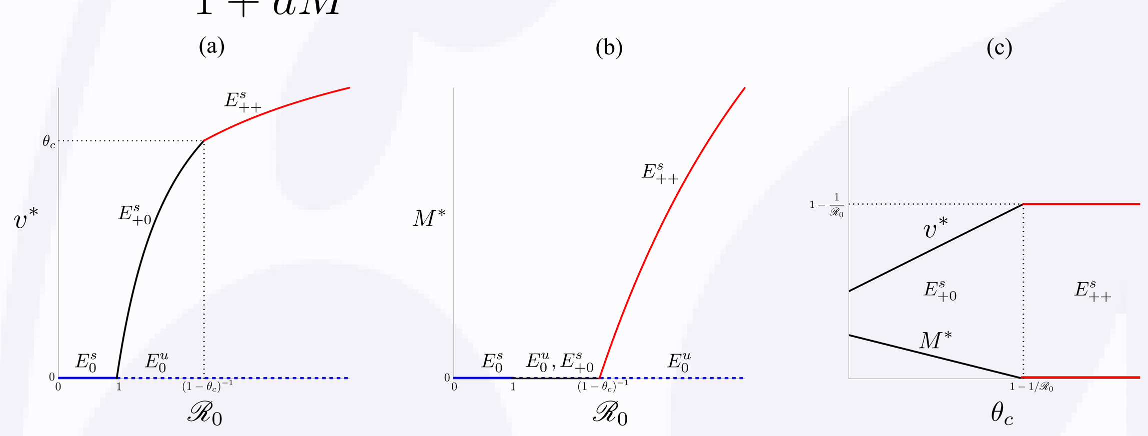

Figure 2. Solid line and curves are for stable equilibria E_0^*, E_{+0}^* and E_{++}^* in (a) and (b), for E_{++}^* when $\theta_c < 1 - 1/\mathscr{R}_0$ and E_{+0}^* when $\theta_c > 1 - 1/\mathscr{R}_0$ in (c). Dashed lines in (a) and (b) are for the unstable equilibrium E_0^u. Numerically drawn with (a,b) $\theta_c = 0.6$; (c) $\mathscr{R}_0 = 2.5$, and commonly $a = 5.0$; $\delta = 10.0$; $\eta = 5.0$.

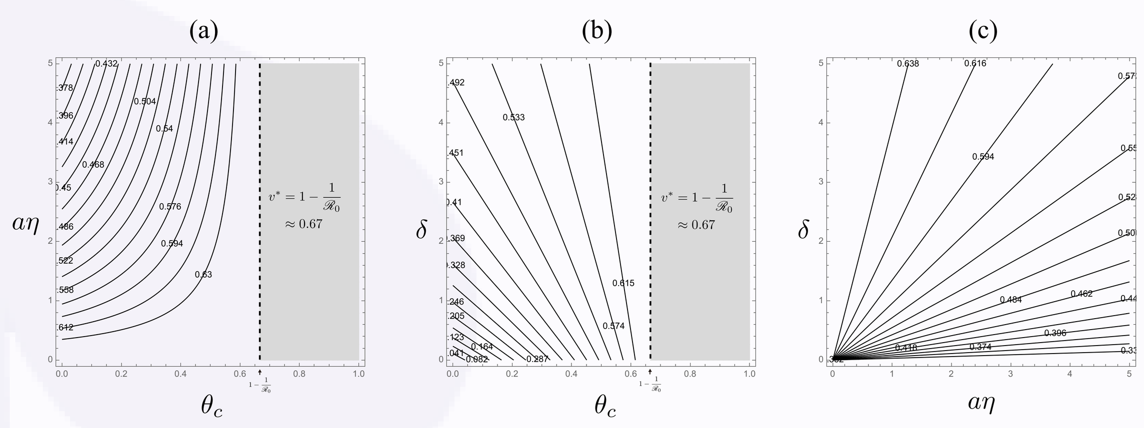

Figure 3. Parameter dependence of the endemic size v^* at the endemic equilibrium which is E_{++} if $\theta_c \le 1 - 1/\mathscr{R}_0$, and E_{+0} if $\theta_c > 1 - 1/\mathscr{R}_0$ respectively. Numerically drawn with (a) $\delta = 2.0$; (b) $a\eta = 5.0$; (c) $\theta_c = 0.3$, and commonly $\mathscr{R}_0 = 3.0$.

Occurrence of a damped oscillation

Theorem 3. When $\mathscr{R}_0 > (1-\theta_c)^{-1}$, the system approaches E_{++} with a damped oscillation if and only if $\theta_-^c < \theta_c < \theta_+^c$, where

$$\theta_\pm^c := \frac{(\delta + a\eta)x_\pm - (\mathscr{R}_0 - 1)\delta}{a\eta(1 + x_\pm)}$$

with

$$x_\pm := \left(\delta + \frac{2a\eta}{\mathscr{R}_0}\right) \pm \sqrt{\left(\delta + \frac{2a\eta}{\mathscr{R}_0}\right)^2 - \delta^2}.$$

If $\theta_c \le \theta_-^c$ or $\theta_c \ge \theta_+^c$, the system approaches an equilibrium in a monotonic manner.

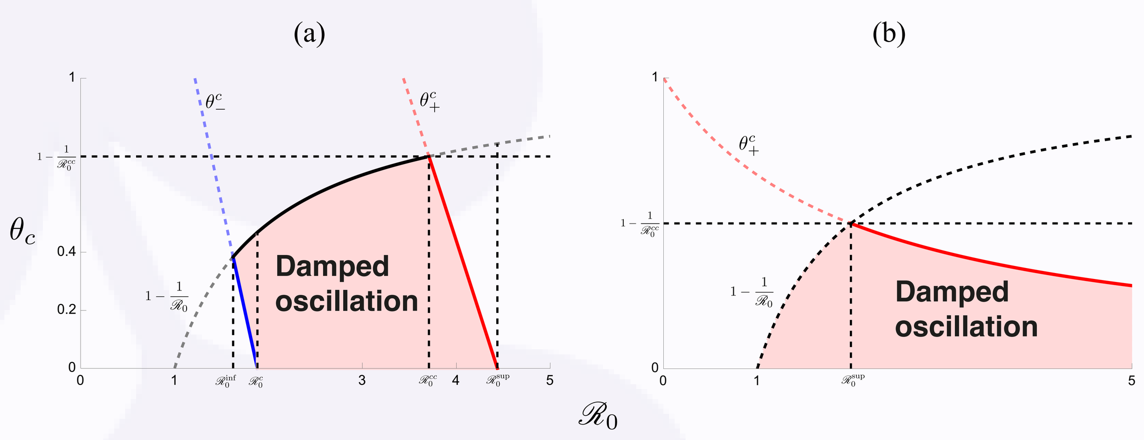

Figure 5. $(\mathscr{R}_0, \theta_c)$-dependence of the occurrence of a damped oscillation around the endemic equilibrium E_{++}. Numerically drawn with (a) $\delta = 1.5$; (b) $\delta = 0$, and commonly $a\eta = 0.5$.

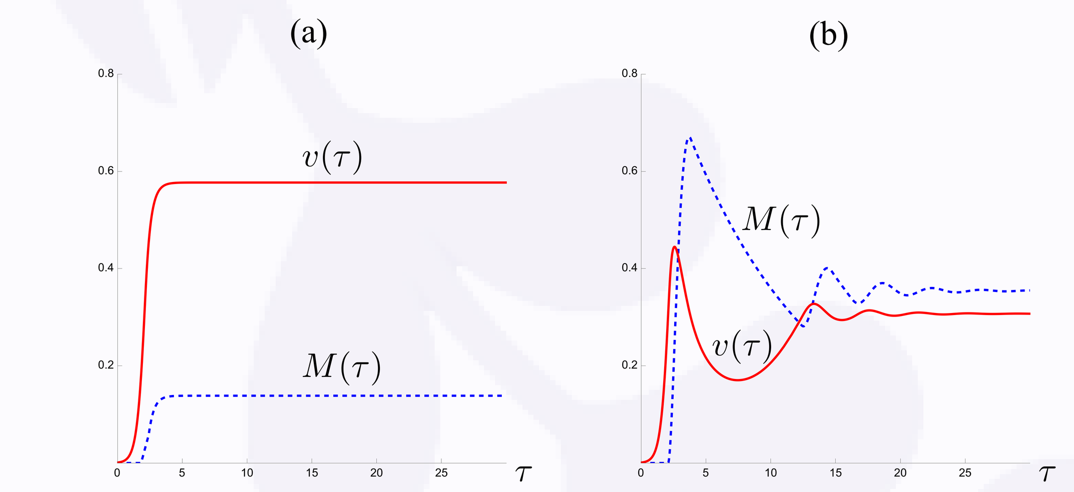

Figure 4. $a = 5.0$; $\mathscr{R}_0 = 4.0$; $\eta = 5.0$; $\theta_c = 0.3$; (a) $\delta = 10.0$; (b) $\delta = 0.1$. The initial condition is commonly given as $(v(0), M(0)) = (0.001, 0.0)$.

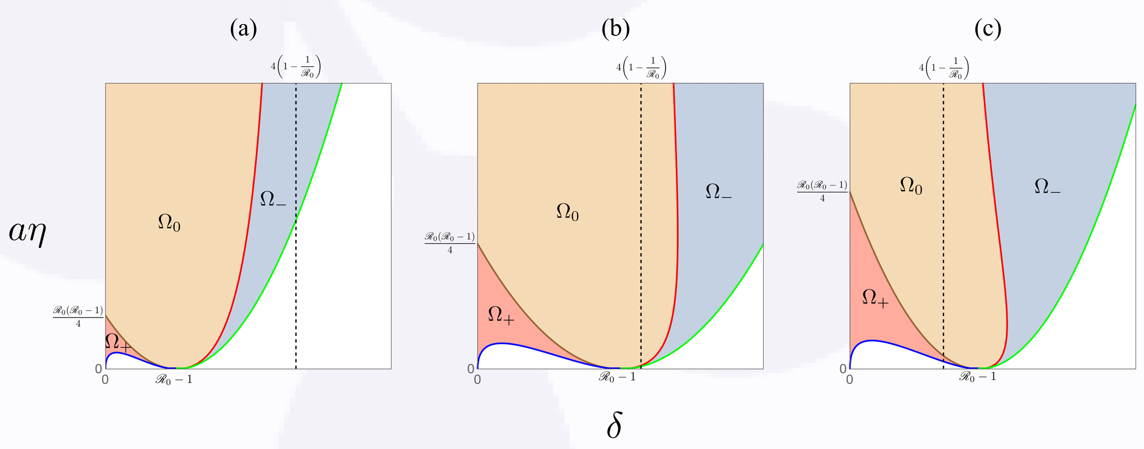

Figure 6. $(\delta, a\eta)$-dependence of the occurrence of a damped oscillation. (a) $1 < \mathscr{R}_0 \le 2$; (b) $2 < \mathscr{R}_0 \le 4$; (c) $\mathscr{R}_0 > 4$. Boundary curves between Ω_- and Ω_0, between Ω_0 and Ω_+, between Ω_+ and blank region, between blank region and Ω_- correspond to $\theta_-^c = 0$, $\theta_+^c = 1 - 1/\mathscr{R}_0$, $\theta_+^c = 0$ and $\theta_-^c = 1 - 1/\mathscr{R}_0$, respectively.

Conclusion

Analysis on our model show that recurring outbreaks occur only when the system approaches an endemic equilibrium at which the social response is maintained. In another endemic case where the social response disappears, the system approaches it in a monotonic manner, that is, the temporal variation of infective population size is monotonic around the endemic equilibrium. For the disease with sufficiently low or sufficiently high transmissibility, the recurring outbreaks of epidemic dynamics is little likely to occur. For the disease with a certain range of transmissibility, the recurring outbreaks are much likely to occur if the community is sufficiently sensitive to the disease spread, or if the social response is sufficiently efficient to reduce the risk of infection. In contrast, the recurring outbreaks may not occur if the community is too insensitive to the disease spread.

*XIE Ying, xie.ying.p4@dc.tohoku.ac.jp