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Fractal Dimensionality of Brain Wave

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Abstract. Fractal natures of brain wave, that is of electroencephalogram (EEG), are investigated, made use of some fractal dimensionalities of it. Especially as for EEG with α-wave, those of frontal and occipital region are compared with respect of the fractal dimension. Significant difference of those dimensions gives such a conjecture that those EEGs with α-wave are independently created. It is indicated that EEG can be modelled by a deterministic dynamical system with a chaotic nature.

1. Introduction

Recently numbers of papers are published on the fractal dimensionality of turbulent time-series of data. One of those time-series data is of brain wave, that is of electroencephalogram (EEG). Although, depending on the method of the fractal analysis on the time-series data and on the experimental condition under which the data are sampled, the obtained fractal dimensions are more or less different from each other, EEG has been an attractive object to be investigated with respect of the fractal nature (ARLE and SIMON, 1990; BABLOYANTZ *et al.*, 1985; MAYER-KRESS and LAYNE, 1987; WATT and HAMEROFF, 1988; XU-NAN and XU-JINGHUA, 1988).

In this paper, EEG, mainly that with α-wave, is investigated with respect of its fractal dimensionality. Since the EEG with α-wave is relatively easy to be distinguished from the others, made use of the power spectrum and the wave form, we select it as the target of our analysis. We have been sampled EEG data through a series of experiments following the international 10–20 system placement (Fig. 1) (REILLY, 1982). After EEG amplitude is recorded with an analog recorder, the data is converted from it to a series of digital valuables with an appropriate time-interval. We shall apply three different approach to analyze the data with respect of the fractal dimensionality: a) correlation dimension; b) dimensionality of cumulative distribution; c) fractal dimension of graph. The approach of correlation dimension has been applied to estimate the fractal nature of a variety of turbulent time-series data (COHEN and PROCACCIA, 1985; GOLDBERGER *et al.*, 1984; HOUQIANG-LI *et al.*, 1990; KARINIEMI and AMMALA, 1981; LIEBOVITCH *et al.*, 1987; MONDANLOU and FREEMAN, 1982;

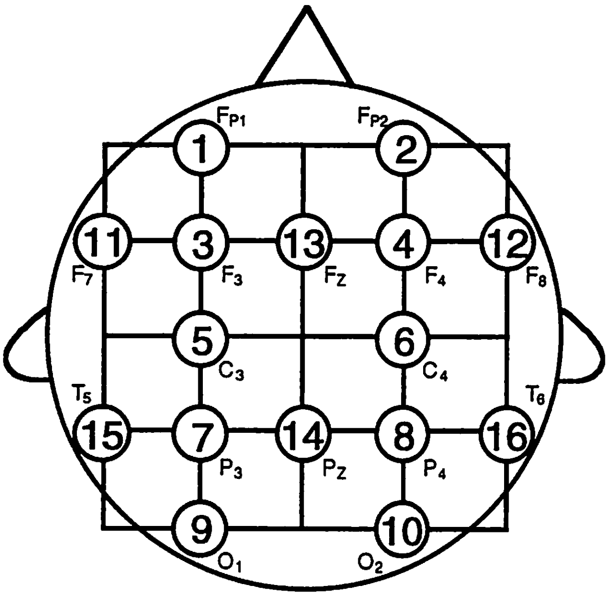

Fig. 1. The international 10–20 system placement to sample EEG.

MPITSOS *et al.*, 1988; PICKOVER and KHORASANI, 1986). In order that the cumulative distribution of a time-variable shows a fractal dimensionality, it must be a geometric distribution. We shall investigate if the dimensionality of cumulative distribution of EEG amplitude is applicable for EEG or not. The third fractal dimensionality we choose, the fractal dimension of graph will be shown to be useful to estimate the fractal nature of EEG. It corresponds to the dimension analysis on the fractal nature of coastal line (HIGUCHI, 1988; MANDELBROT, 1982). These latter two methods have not been applied to the analysis on EEG and our analysis will be one of pioneers about such types of analysis on EEG. Making use of these three approaches, we shall consider the fractal nature of EEG and try to discuss some aspects of cerebral system creating EEG.

2. Analyses and Results

We analyze time-series of EEG data sampled around 30000 temporally successive amplitudes with 2.56 msec time interval, making use of three different approaches to pull out the fractal nature of EEG (as for the concrete calculating method, see Appendix). The number of samples in each data is equal to that of subjects committed to the experiment, that is, every time-series of EEG amplitudes is sampled from each different subject.

2.1. Correlation dimension

For a time-series of data $(x_0, x_1, ..., x_M)$, where x_i $(i = 0, 1, ..., M)$ is the i-th data of EEG amplitude and M is the total number of data obtained with the time interval Δt, we can calculate the following correlation integral:

$$C(r) = \frac{1}{N^2} \sum_{i}^{N} \left\{ \sum_{j}^{N} H\left(r - \left\| \boldsymbol{x}_i - \boldsymbol{x}_j \right\|\right) - 1 \right\},$$

where the function $H(z)$ is the step function which is 1 for non-negative z and 0 for negative z. $\| \cdot \|$ is a proper norm for the d-dimensional space. N is the total number of d-dimensional vectors $\boldsymbol{x}_i$ $(i = 1, 2, ..., M - d + 1)$ which are constructed from $(x_0, x_1, ..., x_M)$ as $\boldsymbol{x}_i = (x_i, x_{i+1}, x_{i+2}, ..., x_{i+d-1})$ $(i = 1, 2, ..., M - d + 1)$ (TAKENS, 1980). If the correlation integral $C(r)$ satisfies

$$C(r) \sim r^D,$$

then the power D can be regarded as a fractal dimension, say "correlation dimension", and it can saturate for a sufficiently large embedded dimension d (GRASSBERGER and PROCACCIA, 1983). It implies the dimension of attractor in the d-dimensional phase space, which is created by a dynamical system governing the time-series. Thus, if we can appropriately estimate the correlation dimension D for EEG, it gives such a possibility that the EEG may be governed by a dynamical system and further may be a low dimensional chaos. Following TAKENS (1980) and GRASSBERGER and PROCACCIA (1983), by the correlation dimension D we can find the number n of independent variables governing the hypothesized but unknown dynamical system: $n = [D] + 1$, where $[D]$ means the largest integer less than or equal to D.

In Table 1, the result for EEG with α-wave, with β-wave, and wave mixed both are shown. The mixed wave seems to have an intermediate correlation dimension, compared with those of EEG with α-wave and that with β-wave, though we could not sample satisfactory data for the case of mixed wave.

On the other hand, we investigated the correlation dimension of EEGs with α-wave respectively of the frontal region (the channels F_3 and F_4 in Fig. 1) and of the occipital one (the channels O_1 and O_2 in Fig. 1), too. For 17 samples of health subjects, it becomes 3.68 $\pm$ 0.25 for the frontal region; 3.47 $\pm$ 0.23 for the occipital region (mean $\pm$ S.D.). Moreover,

Table 1. Correlation dimension of EEG. The number in the bracket shows that of sampled data. The mean $\pm$ standard deviation is shown for the fractal dimension of α-wave and β-wave. In the case of mixed wave, sufficient data could not be sampled.

Sample	Correlation dimension
α-wave	3.58 $\pm$ 0.26 (35)
β-wave	5.52 $\pm$ 0.97 (13)
Mixed wave	4.5 ~ 5.5

in order to show the condition-dependency of the dimension, the EEG with α-wave sampled from the occipital region of the patients with epilepsy were sampled in 13 data and investigated: 3.62 ± 0.25 (mean ± S.D.).

2.2. Dimensionality of cumulative distribution

If the cumulative distribution $P(X)$ for an amplitude X of EEG follows the following geometric law:

$$P(X) \equiv \int_X^\infty p(y)dy \approx X^{-\gamma},$$

then the power γ can be regarded as a fractal dimension derived from the cumulative distribution (MANDELBROT, 1982). Although it has not yet been given any satisfactory explanation why there are some distributions following such a geometric law for natural phenomena, the dimensionality can be regarded as one of strong characteristics of the time-series data, if it can be appropriately estimated.

Although obtained cumulative distribution of EEG for some data sets is partially linear with the log-log axis, it generally seem not to follow any geometric law as a whole. Instead, as shown in Fig. 2, the distribution for some data seems to be alternatively exponential rather than geometric. Since our present interest is to investigate the fractal nature of EEG, we shall not mention anymore if the cumulative distribution of EEG amplitude may be exponential or not. It is open problem which may reveal another feature of EEG. As our result, it is shown that EEG does not have the fractal nature in terms of the cumulative distribution.

2.3. Fractal dimension of graph

Since the temporal variation of EEG is expressed as a 2-dimensional graph, the fractal dimension of the graph can be estimated. We shall apply the estimating method making use of the cumulative length of EEG oscillation. In detail, we deal with the following quantity (HIGUCHI, 1988):

$$L_{\Delta t} \equiv \sum_{i=1}^{M-1} |X_{i+1} - X_i|,$$

which corresponds to the cumulative length of EEG oscillation, measured in a time unit Δt. If $L_{\Delta t} \propto \Delta t^{-\beta}$, then the power β can be regarded as the fractal dimension of EEG graph. If β can be appropriately determined, it indicates a fractal nature of EEG.

The result for EEG with α-wave is shown in Table 2. The dimension is estimated respectively for EEGs with α-wave of frontal region and of occipital one. The dimension is significantly higher in the occipital region than in the frontal. The correlation coefficient between the frontal and the occipital EEGs with α-wave is estimated by the data for each subject. The fractal dimensions of graph for those EEGs with α-wave have a low correlation coefficient. This result indicates that those EEGs with α-wave of the frontal and the occipital regions can be regarded independent each other. In contrast, as shown in Table 2, the spectral

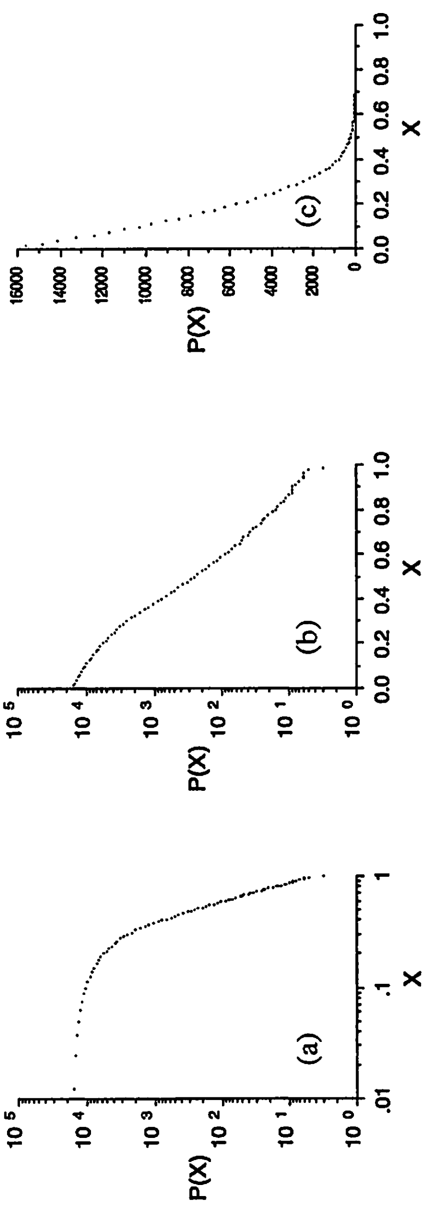

Fig. 2. An example of sampled cumulative cumulative distribution of EEG with α-wave amplitude. (a) log-log axis; (b) normal-log axis; (c) normal-normal axis. Maximum amplitude is normalized to 1.

Table 2. Fractal dimension of graph and spectral fluctuation of α-wave. α-waves of the frontal and the occipital regions are independently investigated. The mean ± standard deviation is shown for the fractal dimension. Correlation coefficient between the data for two regions is calculated, too. 16 sets of data are used.

	Frontal region	Occipital region	Correlation coefficient
Spectral fluctuation	0.94 ± 0.14	1.03 ± 0.14	0.83
Graph dimension	1.83 ± 0.02	1.86 ± 0.03	0.07

fluctuation of EEG amplitude with α-wave shows high correlation between those two regions (as for the analysis of spectral fluctuation, for instance, see KOBAYASHI and MUSHA, 1982; also see Appendix about our calculating way of the spectral fluctuation).

Also in this analysis, we can show the condition-dependency of the dimension, analyzing 13 data of EEG with α-wave with epilepsy: 1.65 ± 0.13 (mean ± S.D.).

3. Discussion

Fractal nature of EEG is investigated to show such a possibility that EEG may be able to be described as a chaotic oscillation driven by a dynamical system.

The result of our analysis on fractal dimensionality of EEG is got together in Fig. 3. Consequently, we conclude that EEG has a clear fractal nature and thus can be described or at least can be modelled by a deterministic dynamical system with a finite number of variables.

By our fractal analysis on EEG with α-wave, it is concluded that EEG with α-wave of the frontal region is correlated little with that of the occipital one. This implies that there may be two systems driving EEG with α-wave, which have a weak connectivity between them. Contrarily, since the spectral fluctuation results in a high correlation between the frontal and the occipital regions, it is probable that the spectral fluctuation may be originated either to a system different from the system driving EEG with α-wave or to a part common between those two systems driving EEG with α-wave. Provided that the α-rhythm would have its origin in an activity of hypothalamus, the resulted difference of EEG fractal nature between the frontal and the occipital regions can be regarded to show the different type of modification of the activity in hypothalamus until it appears as EEG sampled on the scalp. Hence, it is conjectured that there may be two systems modifying the α-rhythm activity in hypothalamus, and that the spectral fluctuation may be origined in the system creating the α-rhythm in hypothalamus.

Although the sampled data are not sufficient up to now, the result that the correlation dimension of (α, β)-mixed wave is intermediate between those of EEGs with α-wave and with β-wave leads to the following possibility: EEG with β-wave may be driven by the system same with that driving EEG with α-wave.

Resulted correlation dimensions and graph dimensions seem not to have the similar tendency in terms of recording site (Fig. 3). For example, the dimension is higher in the frontal region than in the occipital in case of correlation dimension, while higher in the

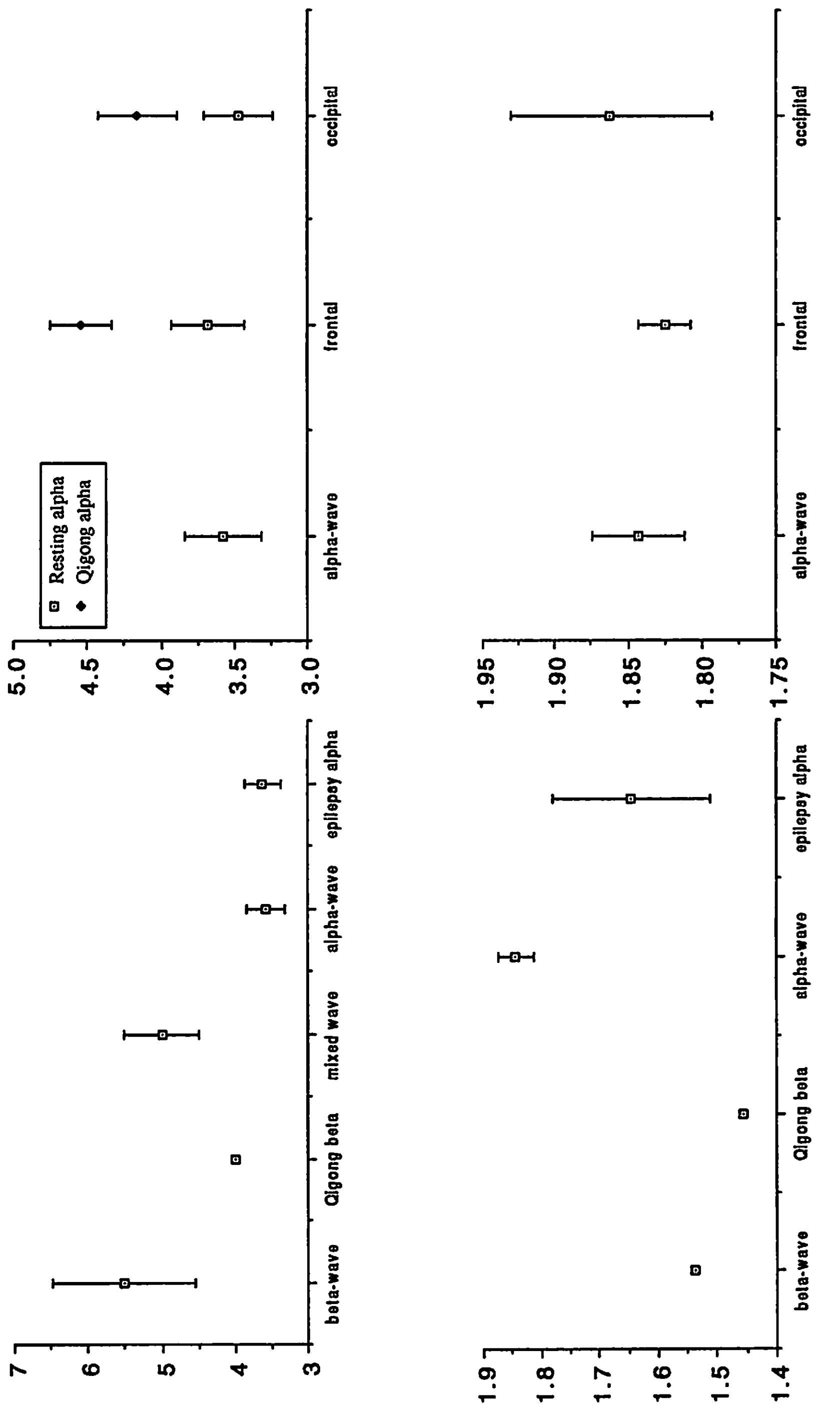

Fig. 3. Resulted correlation dimensions and graph dimensions of EEG. Error bars are for standard deviation except for the case of EEG with β-wave, in which the bar indicates the standard error. Without error bars, the number of samples is so small that those points are for examples.

occipital region in case of graph dimension. This shows such a possibility that these two dimensions may express two different aspects of EEG.

The quantitative results by our analysis will be possibly different more or less from those in previous reports and forecoming ones. For example, MAYER-KRESS and LAYNE (1987) results that the correlation dimension of EEG sampled from occipital leads in awake and eyes closed state is 4–7. This variation of resulted dimension may be because the fractal dimension strongly depends on the method to calculate it by data and is influenced by the quality of data very much, and because EEG is very sensitive to the condition in and out of the subject in the experiment. Indeed, our analysis for the data under some different conditions results in fractal dimensions different from each other among them. Further, for seven data of EEG with α-wave sampled from some subjects resting in Qigong state, where Qigong is a Chinese therapeutic training of Indian Yoga type, the analysis results in the higher correlation dimension than that for the resting EEG with α-wave: 4.54 ± 0.21 for the frontal region; 4.16 ± 0.27 for the occipital region (mean ± S.D.) (Fig. 3). XU-NAN and XU-JINGHUA (1988) also mentions that the correlation dimension of EEG changes in the Qigong state.

Lastly, it is the difficulty of fractal analysis on EEG that the quantitative result normally includes some noises and is sensitively affected by the condition under which the data are sampled. On the other side, we remark that, if we will be able to estimate a finite value of correlation dimension of EEG by the accumulation of researches on it, even though the result might have a range of variation, it indicates the possibility of modelling EEG dynamics by a dynamical system with a finite number of variables. This means that EEG dynamics might be essentially driven by an appropriate small number of factors instead of such a large number of factors as to be required some statistical treatments for its description. As for the dimension of graph, since its calculation does not require so long time with personal computers, its application not only for the investigation on the fractal nature of EEG but also for the detection of trasients in EEG will be expected (see ARLE and SIMON, 1990).

We hope that our work will contribute to the promotion of various researches on EEG.

APPENDIX: Methods to calculate fractal dimensions from data

In this appendix, it is shown how the calculation is carried out for each of three methods applied to EEG data in order to estimated the fractal dimensionality.

Correlation Dimension: For d-dimensional embedded vectors constructed from EEG data as mentioned in the main text, we calculate the number $\#_r(x_{k_j})$ of vectors in a distance r from a randomly selected vector x_{k_j} determined by a randomly selected integer k_j ($j = 1, 2, ..., L$) less than $N + 1$. L is the number of randomly selected vectors which are used to calculate this value. L should be sufficiently large. Then the correlation integral $C(r)$ corresponds to

$$\frac{\sum_{j=1}^{L} \#_r\left(x_{k_j}\right) / N^2}{L},$$

Therefore if $C(r) \sim r^D$, we can estimate the correlation dimension D from the linear relation, that is the slope of graph, between $\log r$ and $\log \sum_{j=1}^{L} \#_r(x_{k_j})$.

Dimensionality of Cumulative Distribution: It is necessary only to count the number of data of EEG amplitude more than X in order to obtain the cumulative cumulative distribution $P(X)$ for an amplitude X from the data. Actually,

$$P(X) \approx \frac{\sum_{i=1}^{M} H(X - x_i)}{M},$$

where the function $H(z)$ is 1 for non-negative z and 0 otherwise. M is the total number of data. If the relation $P(X) \propto X^{-\gamma}$ is satisfied, the power γ is determined from the slope of graph between $\log X$ and $\log \sum_{i=1}^{M} H(X - x_i)$.

Fractal Dimension of Graph: The value $L_{\Delta t}$ can be easily calculated from the data for a time-interval Δt. Then, with the same data set for Δt, we can calculate $L_{2\Delta t}$, $L_{3\Delta t}$, and so on, making use of the following correspondence:

$$\begin{aligned}
\Delta t &\leftrightarrow \{x_1, x_2, x_3, \ldots, x_M\} \\
2\Delta t &\leftrightarrow \{x_1, x_3, x_5, \ldots\} \\
& \{x_2, x_4, x_6, \ldots\} \\
3\Delta t &\leftrightarrow \{x_1, x_4, x_7, \ldots\} \\
& \{x_2, x_5, x_8, \ldots\} \\
& \{x_3, x_6, x_9, \ldots\}
\end{aligned}$$

Therefore, if $L_{\Delta t} \propto \Delta t^{-\beta}$, the power β can be estimated from the slope of graph between $\log n$ and $\log \sum L_{n\Delta t}$, where the sum $\sum$ is carried over n corresponding sets for an given n as shown above.

Spectral Fluctuation: The spectral fluctuation analysis is to analyze the data of brain wave with respect of its spectrum. Although the brain wave has a various elements of its own fluctuation: the fluctuation of its amplitude; of its period; of its intercepts with the zero level, etc. We investigate the EEG with α-wave's spectrum in terms of its intercepts with the zero level. The concrete analyzing way is as follows:

(a) the spectrum of an EEG with α-wave data is calculated;

(b) the α-region (8–10 Hz) is sampled apart from the data of the other region;

(c) from the isolated data, the wave graph is reconstructed;

(d) the lengths of intervals are sampled in turn between two neighbor intercepts with the zero level of the graph;

(e) with those sampled lengths in order, the FFT (Fast Fourier Transformation) is applied and the spectrum is obtained;

(f) the obtained spectrum is fitted in the log-log axis by a linear line and the slope is sampled. This slope can be regarded as an index of spectral fluctuation of the EEG with α-wave.

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