Analysis of a mathematical model on the modification of apparent competition effect with the invasion of alien predator

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Making use of Lotka–Volterra prey–predator system (1), we investigated the transition of equilibrium state with the modification of apparent competition between preys due to the invasion of an alien predator into 2 prey–1 predator system. We consider the system (1) of two predators and two preys. P_i is the population density of predator i, H_i the population density of prey i, δ_i the natural death rate of predator i, r_i the intrinsic growth rate of prey i, c_{ij} the energy conversion coefficient for the predator i to predate prey j, b_{ij} the predation coefficient for prey j to be predated by the predator j, β_i the coefficient of intra-specific competition for prey i. In this model, there is no inter-specific interaction between two preys. Only predation relation is involved in it. Since two preys have common predators, we can say that they are in an apparent competition. Moreover, there is no direct interaction between predators which have common preys so that they are in an exploitative competition. We do not assume (i.e., do ignore) the intra-specific density effect for the predator. We assume that $0 < H_i(0) \leq r_i/\beta_i$ $(i = 1, 2)$ as the initial condition for the prey, where r_i/β_i is the carring capacity for prey i. As a mathematical model, such condition that the initial value $H_i(0)$ does not exceed the carring capacity is naturally required. Our analysis can demonstrate the case that the endangered species could be rescued and recovered with the introduction of an alien predator. Moreover, by the introduction of an apporopriate alien predator, it would be possible to make a target species go extinct.

Lotka–Volterra – – 1

Lotka–Volterra –

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\begin{cases}
\dfrac{dP_1}{dt} &= -\delta_1 P_1 + c_{11}b_{11}H_1 P_1 + c_{12}b_{21}H_2 P_1 \\[2mm]
\dfrac{dP_2}{dt} &= -\delta_2 P_2 + c_{21}b_{12}H_1 P_2 + c_{22}b_{22}H_2 P_2 \\[2mm]
\dfrac{dH_1}{dt} &= (r_1 - \beta_1 H_1)H_1 - b_{11}H_1 P_1 - b_{12}H_1 P_2 \\[2mm]
\dfrac{dH_2}{dt} &= (r_2 - \beta_2 H_2)H_2 - b_{21}H_2 P_1 - b_{22}H_2 P_2
\end{cases}
\tag{1}
$$

P_i i H_i i δ_i i r_i i

c_{ij} i j b_{ij} j

i β_i i 2

2

2

( )

$0 < H_i(0) \leq r_i/\beta_i \ (i = 1, 2)$ r_i/β_i i

$H_i(0)$