

The homogenization method for topology optimization of structures: old and new

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Topology optimization of structures is nowadays a well developed field with many different approaches and a wealth of applications. One of the earliest method of topology optimization was the so-called homogenization method, introduced in the early eighties. It became extremely popular in its over-simplified version, called SIMP (Solid Isotropic Material with Penalisation), which retains only the notion of material density and forgets about true composite materials with optimal (possibly non isotropic) microstructures. However, the appearance of mature additive manufacturing technologies which are able to build finely graded microstructures (sometime called lattice materials) drastically change the picture and one can see a resurrection of the homogenization method for such applications. Indeed, homogenization is the right technique to deal with microstructured materials where anisotropy plays a key role, a feature which is absent from SIMP. Homogenization theory allows to replace the microscopic details of the structure (typically a complex networks of bars, trusses and plates) by a simpler effective elasticity tensor describing the mesoscopic properties of the structure.

The goal of this course is to review the necessary mathematical tools of homogenization theory and apply them to topology optimization of mechanical structures. The ultimate application, targeted in this course, is the topology optimization of structures built with lattice materials. Practical and numerical exercices will be given, based on the finite element free software FreeFem++.

References

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