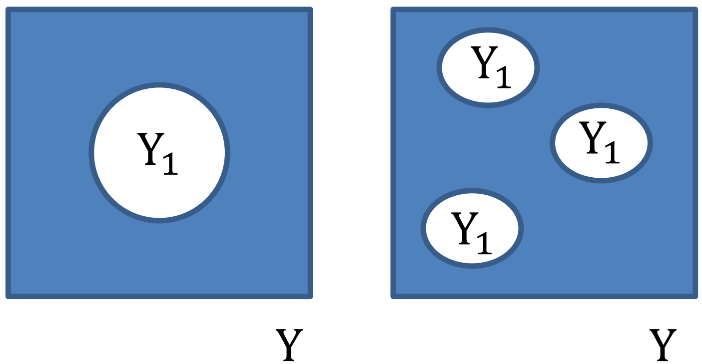
Lecture of FreeFEM++

Update:2018/3/24

homogenization

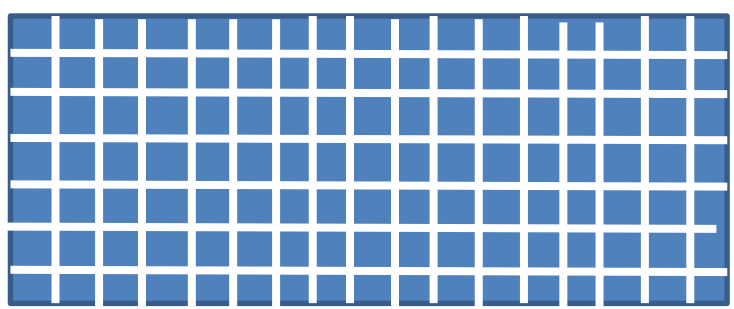
- Micro Scale:



High Diffusion in Y_1

Low Diffusion in $Y \setminus Y_1$

- Macro Scale:
  - Micro Scale domain exist at regular interval
  - Numerical results depend on the shape of Y_1 of Micro scale.



Algorithm

- Step 1
  - Numerical Calculation in Micro Scale $\mathrm{Y} = (y_1, y_2)$ with periodic B.C.,
- Step 2
  - Estimation of Tensor A_{eff} for diffusion term
- Step 3
  - Numerical Calculation in Macro Scale in $\Omega = (x_1, x_2)$

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Step 1

- Numerical Calculation in Micro Scale $\mathrm{Y} = (y_1, y_2)$ with periodic B.C.,

$$\int_{\mathrm{Y}} A\nabla_y w_i \cdot \nabla_y W_i \, dy = -\int_{\mathrm{Y}} Ae_i \cdot \nabla_y W_i \, dy,$$

$$\frac{1}{|\mathrm{Y}|}\int_{\mathrm{Y}} w_i \, dy = 0$$

$$(i = 1) \Rightarrow \int_{\mathrm{Y}} A\left(\frac{\partial w_1}{\partial y_1}\frac{\partial W_1}{\partial y_1} + \frac{\partial w_1}{\partial y_1}\frac{\partial W_1}{\partial y_1}\right) dy = -\int_{\mathrm{Y}} A\frac{\partial W_1}{\partial y_1} dy$$

$$\Rightarrow w_1 := w_1 - \frac{1}{|\mathrm{Y}|}\int_{\mathrm{Y}} w_1 \, dy$$

$$(i = 2) \Rightarrow \int_{\mathrm{Y}} A\left(\frac{\partial w_2}{\partial y_1}\frac{\partial W_2}{\partial y_1} + \frac{\partial w_2}{\partial y_1}\frac{\partial W_2}{\partial y_1}\right) dy = -\int_{\mathrm{Y}} A\frac{\partial W_2}{\partial y_2} dy$$

$$\Rightarrow w_2 := w_2 - \frac{1}{|\mathrm{Y}|}\int_{\mathrm{Y}} w_2 \, dy$$

Algorithm

- Step 1
  - Numerical Calculation in Micro Scale $\mathrm{Y} = (y_1, y_2)$ with periodic B.C.,

- Step 2
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- Step 3
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Step 2

- Estimation of Tensor A_{eff} for diffusion term

$$A_{\mathrm{eff}} = \begin{bmatrix} \alpha_{1,1}(w_1, w_2) & \alpha_{2,1}(w_1, w_2) \\ \alpha_{1,2}(w_1, w_2) & \alpha_{2,2}(w_1, w_2) \end{bmatrix}$$

$$\alpha_{1,1} = \int_{\mathrm{Y}} A \left\{ 1 + 2\frac{\partial w_1}{\partial y_1} + \left(\frac{\partial w_1}{\partial y_1}\right)^2 + \left(\frac{\partial w_1}{\partial y_2}\right)^2 \right\} dy$$

$$\alpha_{2,2} = \int_{\mathrm{Y}} A \left\{ 1 + 2\frac{\partial w_2}{\partial y_2} + \left(\frac{\partial w_2}{\partial y_1}\right)^2 + \left(\frac{\partial w_2}{\partial y_2}\right)^2 \right\} dy$$

$$\alpha_{1,2} = \alpha_{2,1} = \int_{\mathrm{Y}} A \left\{ \frac{\partial w_2}{\partial y_1} + \frac{\partial w_1}{\partial y_2} \right\} dy$$

Algorithm

- Step 1
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Step 3

- Numerical Calculation in Macro Scale in $\Omega = (x_1, x_2)$

$$\int_{\Omega} A_{\text{eff}} \nabla_x u \cdot \nabla_x v \, dx - \int_{\Omega} f v \, dx = 0$$

$$\Rightarrow \int_{\Omega} \begin{pmatrix} \alpha_{1,1} \frac{\partial u}{\partial x_1} \frac{\partial v}{\partial x_1} + \alpha_{2,2} \frac{\partial u}{\partial x_2} \frac{\partial v}{\partial x_2} \\ + \alpha_{1,2} \frac{\partial u}{\partial x_2} \frac{\partial v}{\partial x_1} + \alpha_{1,2} \frac{\partial u}{\partial x_1} \frac{\partial v}{\partial x_2} \end{pmatrix} dx$$

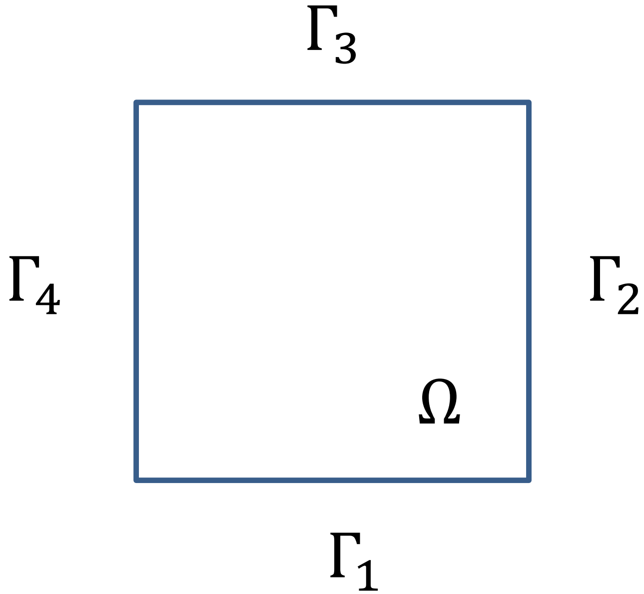
$$- \int_{\mathrm{Y}} f v \, dx = 0$$

Homogenization.edp

- Micro Scale:

$$A = 0.5 + 0.5 * \{\sin(4y_1\pi)\cos(2y_2\pi)\}$$

- Macro Scale:



$$\int_\Omega A_{\text{eff}} \nabla_x u \cdot \nabla_x v \, dx = 0 \text{ in } \Omega,$$
$$u = 0 \text{ on } \Gamma_1 \cap \Gamma_3$$
$$\frac{\partial u}{\partial v} = 0 \text{ on } \Gamma_2$$
$$u = 1 \text{ on } \Gamma_4$$

Problem 26

- Change A in Homogenization.edp,
- Check the value of “outflow”

Special Problem

Micro Scale-Topology Optimization

Find θ under the volume preserving

- The cost function:

$$F(u) = \frac{1}{2}\int_{\Omega} u^2 \, dx$$

- The constraint function:

$$G_1(\phi, u, v, w_1, w_2) = \int_{\Omega} A_{\text{eff}} \nabla_x u \cdot \nabla_x v \, dx - \int_{\Omega} f v \, dx,$$

$$G_2(\phi, w_1, w_2, W_1, W_2)$$

$$= \sum_{i=1}^{2} \left\{ \int_{\mathrm{Y}} \psi(\phi^{\gamma}) \nabla_y w_i \cdot \nabla_y W_i \, dy + \int_{\mathrm{Y}} \psi(\phi^{\gamma}) e_i \cdot \nabla_y W_i \, dy \right\},$$

$$G_3(w_1, w_2) = \sum_{i=1}^{2} \left\{ \frac{1}{|\mathrm{Y}|} \int_{\mathrm{Y}} w_i \, dy \right\}$$

for real valued parameters $\alpha, \beta, \gamma, \psi_1$

$\phi = 0.5\{\tanh(\beta\theta) + 1\}$ and $\psi(\phi^{\gamma}) = \alpha_1(1 - \phi^{\gamma}) + \alpha_2 \phi^{\gamma}$

Cont. Special Problem

$$L(\theta, u, v, w_1, w_2, W_1, W_2) =$$
$$F(u) - G_1(\phi, u, v, w_1, w_2)$$
$$-G_2(\phi, w_1, w_2, W_1, W_2) - G_3(w_1, w_2)$$

Main Problem 1

$$L(\theta, u, v, w_1, w_2, W_1, W_2)[v'] = -\int_\Omega A_{\mathrm{eff}} \nabla_x u \cdot \nabla_x v' \, dx - \int_\Omega f v' \, dx$$

Adjoint Problem 1

$$L(\theta, u, v, w_1, w_2, W_1, W_2)[u'] = \int_\Omega u u' \; dx - \int_\Omega A_{\mathrm{eff}} \nabla_x u' \cdot \nabla_x v \, dx$$

Cont. Special Problem

Main Problem 2

$$L(\theta, u, v, w_1, w_2, W_1, W_2)[W_1', W_2'] = -\sum_{i=1}^{2}\left\{\int_{\mathrm{Y}} \psi(\phi^{\gamma}) \nabla_y w_i \cdot \nabla_y W_i' \, dy + \int_{\mathrm{Y}} \psi(\phi^{\gamma}) e_i \cdot \nabla_y W_i' \, dy\right\}$$

Adjoint Problem 2

$$L(\theta, u, v, w_1, w_2, W_1, W_2)[w_1', w_2'] = -\sum_{i=1}^{2}\left\{\begin{array}{l} \displaystyle\int_{\Omega} \begin{bmatrix} \dfrac{\partial \alpha_{1,1}}{\partial w_i} & \dfrac{\partial \alpha_{1,2}}{\partial w_i} \\ \dfrac{\partial \alpha_{2,1}}{\partial w_i} & \dfrac{\partial \alpha_{2,2}}{\partial w_i} \end{bmatrix} \nabla_x u \cdot \nabla_x v \, dx \\ \displaystyle + \int_{\mathrm{Y}} \psi(\phi^{\gamma}) \nabla_y w_i' \cdot \nabla_y W_i \, dy - \frac{1}{|\mathrm{Y}|}\int_{\mathrm{Y}} w_i' \, dy \end{array}\right\}$$

Cont. Special Problem

$$\frac{\partial \alpha_{1,1}}{\partial w_1} = \int_{\mathrm{Y}} \psi(\phi^{\gamma}) \left\{ 1 + 2\frac{\partial w_1'}{\partial y_1} + 2\frac{\partial w_1}{\partial y_1}\frac{\partial w_1'}{\partial y_1} + 2\frac{\partial w_1}{\partial y_2}\frac{\partial {w_1}'}{\partial y_2} \right\} dy$$

$$\frac{\partial \alpha_{2,2}}{\partial w_1} = 0, \frac{\partial \alpha_{2,1}}{\partial w_1} = \frac{\partial \alpha_{1,2}}{\partial w_1} = \int_{\mathrm{Y}} A\frac{\partial w_1'}{\partial y_2} dy$$

$$\frac{\partial \alpha_{2,2}}{\partial w_2} = \int_{\mathrm{Y}} \psi(\phi^{\gamma}) \left\{ 1 + 2\frac{\partial w_2'}{\partial y_1} + 2\frac{\partial w_2}{\partial y_1}\frac{\partial w_2'}{\partial y_1} + 2\frac{\partial w_2}{\partial y_2}\frac{\partial w_2'}{\partial y_2} \right\} dy$$

$$\frac{\partial \alpha_{1,1}}{\partial w_2} = 0, \frac{\partial \alpha_{2,1}}{\partial w_2} = \frac{\partial \alpha_{1,2}}{\partial w_2} = \int_{\mathrm{Y}} A\frac{\partial w_2'}{\partial y_1} dy$$

Cont. Special Problem

The sensitivity

$$L(\theta, u, v, w_1, w_2, W_1, W_2)[\theta'] = -\int_{\Omega} \left\{ \begin{bmatrix} \frac{\partial \alpha_{1,1}}{\partial \theta} & \frac{\partial \alpha_{1,2}}{\partial \theta} \\ \frac{\partial \alpha_{2,1}}{\partial \theta} & \frac{\partial \alpha_{2,2}}{\partial \theta} \end{bmatrix} \nabla_x u \cdot \nabla_x v \right\} dx - \sum_{i=1}^{2} \left[\int_{\mathrm{Y}} \left\{ (\alpha_2 - \alpha_1) \gamma \phi^{\gamma-1} \frac{\partial \phi}{\partial \theta} \nabla_y w_i \cdot \nabla_y W_i + e_i \cdot \nabla_y W_i \right\} \theta' dy \right]$$

$$\frac{\partial \alpha_{1,1}}{\partial \theta} = -\int_{\mathrm{Y}} (\alpha_2 - \alpha_1) \gamma \phi^{\gamma-1} \frac{\partial \phi}{\partial \theta} \theta' \left\{ 1 + 2 \frac{\partial w_1}{\partial y_1} + \left(\frac{\partial w_1}{\partial y_1} \right)^2 + \left(\frac{\partial w_1}{\partial y_2} \right)^2 \right\} dy$$

$$\frac{\partial \alpha_{2,2}}{\partial \theta} = -\int_{\mathrm{Y}} (\alpha_2 - \alpha_1) \gamma \phi^{\gamma-1} \frac{\partial \phi}{\partial \theta} \theta' \left\{ 1 + 2 \frac{\partial w_2}{\partial y_2} + \left(\frac{\partial w_2}{\partial y_1} \right)^2 + \left(\frac{\partial w_2}{\partial y_2} \right)^2 \right\} dy$$

$$\frac{\partial \alpha_{1,2}}{\partial \theta} = \frac{\partial \alpha_{2,1}}{\partial \theta} = -\int_{\mathrm{Y}} (\alpha_2 - \alpha_1) \gamma \phi^{\gamma-1} \frac{\partial \phi}{\partial \theta} \theta' \left\{ \frac{\partial w_2}{\partial y_1} + \frac{\partial w_1}{\partial y_2} \right\} dy$$