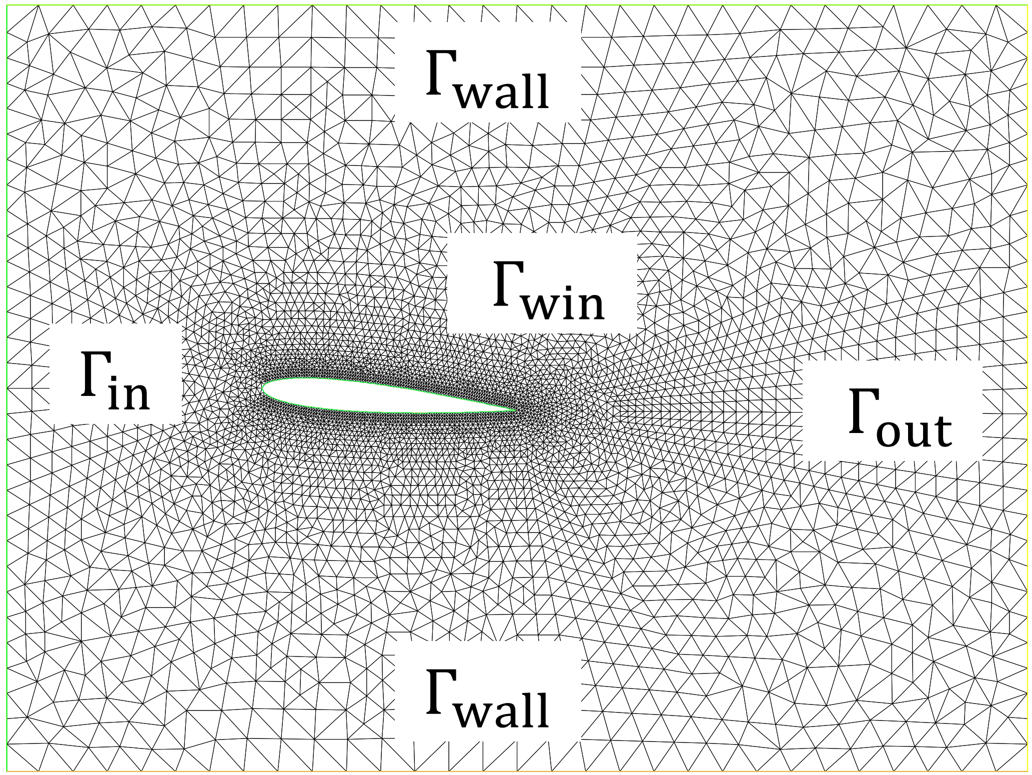
Lecture of FreeFEM++

Update:2017/11/30

Non stationary Navier-Stokes equation(AOA_aerofoil.edp)
非定常Navier-Stokes方程式



$$\frac{\partial \boldsymbol{u}}{\partial t} + (\boldsymbol{u} \cdot \nabla)\boldsymbol{u} = -\nabla p + \frac{1}{\mathrm{Re}} \Delta \boldsymbol{u} \text{ in } \Omega$$

$$\nabla \cdot \boldsymbol{u} = 0 \text{ in } \Omega$$

$$u = 0, v = 0 \text{ on } \Gamma_{\mathrm{wall}} \cup \Gamma_{\mathrm{win}}$$

$$u = -(y + 1.5)(y - 1.5)/1.5^2, v = 0 \text{ on } \Gamma_{\mathrm{in}}$$

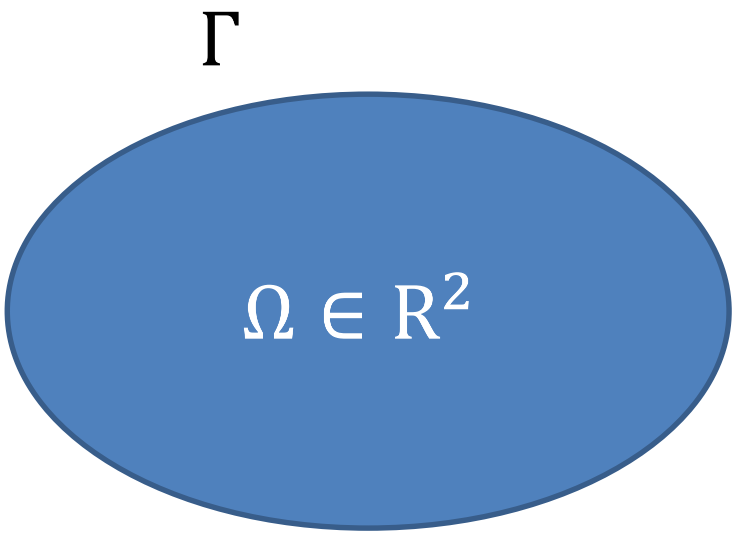
$$\left(p\boldsymbol{n} - \frac{1}{\mathrm{Re}}(\nabla \boldsymbol{u}^{\mathbf{T}})\boldsymbol{n}\right) = \mathbf{0} \text{ on } \Gamma_{\mathrm{out}}$$

Velocity; $\boldsymbol{u} = (u, v)$, Pressure p

Reminder

Stokes equations

- Let trial function for $\boldsymbol{u}$ and p be $\boldsymbol{v}$ and q, where $\boldsymbol{u} = (u_x, u_y)$, $\boldsymbol{v} = (v_x, v_y)$.
- Describe the following equation in weak form.



$$\nabla p - \frac{1}{\mathrm{Re}} \Delta \boldsymbol{u} = 0 \text{ in } \Omega$$

$$\boldsymbol{\nabla} \cdot \boldsymbol{u} = 0 \text{ in } \Omega$$

$$\boldsymbol{u} = \boldsymbol{0} \text{ on } \Gamma$$

Reminder

$$\nabla p - \frac{1}{\mathrm{Re}}\Delta \boldsymbol{u} = 0, \nabla \cdot \boldsymbol{u} = 0$$

$$\Longrightarrow \int_{\Omega} \left\{ \left(\nabla p - \frac{1}{\mathrm{Re}}\Delta \boldsymbol{u} \right) \cdot \boldsymbol{v} - (\nabla \cdot \boldsymbol{u}) q \right\} d\Omega = 0$$

$$\Longrightarrow \int_{\Omega} \begin{bmatrix} \left\{ \frac{\partial p}{\partial x} - \frac{1}{\mathrm{Re}} \left(\frac{\partial^2 u_x}{\partial x^2} + \frac{\partial^2 u_x}{\partial y^2} \right) \right\} v_x \\ + \left\{ \frac{\partial p}{\partial y} - \frac{1}{\mathrm{Re}} \left(\frac{\partial^2 u_y}{\partial x^2} + \frac{\partial^2 u_y}{\partial y^2} \right) \right\} v_y \\ -(\nabla \cdot \boldsymbol{u}) q \end{bmatrix} d\Omega = 0$$

Reminder

$$\Rightarrow \int_\Omega \begin{bmatrix} \dfrac{\partial p}{\partial x} v_x + \dfrac{\partial p}{\partial y} v_y \\ + \dfrac{1}{\mathrm{Re}} \left(\dfrac{\partial u_x}{\partial x} \dfrac{\partial v_x}{\partial x} + \dfrac{\partial u_x}{\partial y} \dfrac{\partial v_x}{\partial y} \right) - \dfrac{1}{\mathrm{Re}} \nabla \cdot \left(\dfrac{\partial u_x}{\partial x} v_x, \dfrac{\partial u_x}{\partial y} v_x \right) \\ + \dfrac{1}{\mathrm{Re}} \left(\dfrac{\partial u_y}{\partial x} \dfrac{\partial v_y}{\partial x} + \dfrac{\partial u_y}{\partial y} \dfrac{\partial v_y}{\partial y} \right) - \dfrac{1}{\mathrm{Re}} \nabla \cdot \left(\dfrac{\partial u_y}{\partial x} v_y, \dfrac{\partial u_y}{\partial y} v_y \right) \\ -(\nabla \cdot \boldsymbol{u}) q \end{bmatrix} d\Omega = 0$$

$$\Rightarrow \int_\Omega \begin{bmatrix} \dfrac{\partial p}{\partial x} v_x + \dfrac{\partial p}{\partial y} v_y \\ + \dfrac{1}{\mathrm{Re}} \left(\dfrac{\partial u_x}{\partial x} \dfrac{\partial v_x}{\partial x} + \dfrac{\partial u_x}{\partial y} \dfrac{\partial v_x}{\partial y} \right) \\ + \dfrac{1}{\mathrm{Re}} \left(\dfrac{\partial u_y}{\partial x} \dfrac{\partial v_y}{\partial x} + \dfrac{\partial u_y}{\partial y} \dfrac{\partial v_y}{\partial y} \right) \\ -(\nabla \cdot \boldsymbol{u}) q \end{bmatrix} d\Omega - \int_\Gamma \frac{1}{\mathrm{Re}} \left(\frac{\partial u_x}{\partial n} v_x + \frac{\partial u_y}{\partial n} v_y \right) d\gamma = 0$$

Reminder

$$\Rightarrow \int_{\Omega} \begin{bmatrix} +\dfrac{\partial}{\partial x}(pv_x) - \dfrac{\partial v_x}{\partial x}p \\ +\dfrac{\partial}{\partial y}(pv_y) - \dfrac{\partial v_y}{\partial y}p \\ +\dfrac{1}{\mathrm{Re}}\left(\dfrac{\partial u_x}{\partial x}\dfrac{\partial v_x}{\partial x} + \dfrac{\partial u_x}{\partial y}\dfrac{\partial v_x}{\partial y}\right) \\ +\dfrac{1}{\mathrm{Re}}\left(\dfrac{\partial u_y}{\partial x}\dfrac{\partial v_y}{\partial x} + \dfrac{\partial u_y}{\partial y}\dfrac{\partial v_y}{\partial y}\right) \\ -(\nabla \cdot \boldsymbol{u})q \end{bmatrix} d\Omega - \int_{\Gamma} \frac{1}{\mathrm{Re}}\left(\frac{\partial u_x}{\partial n}v_x + \frac{\partial u_y}{\partial n}v_y\right) d\gamma = 0$$

$$\Rightarrow \int_{\Omega} \begin{bmatrix} -(\nabla \cdot \boldsymbol{v})p \\ +\dfrac{1}{\mathrm{Re}}\left(\dfrac{\partial u_x}{\partial x}\dfrac{\partial v_x}{\partial x} + \dfrac{\partial u_x}{\partial y}\dfrac{\partial v_x}{\partial y}\right) \\ +\dfrac{1}{\mathrm{Re}}\left(\dfrac{\partial u_y}{\partial x}\dfrac{\partial v_y}{\partial x} + \dfrac{\partial u_y}{\partial y}\dfrac{\partial v_y}{\partial y}\right) \\ -(\nabla \cdot \boldsymbol{u})q \end{bmatrix} d\Omega$$

$$+ \int_{\Gamma} \left\{(pv_x + pv_y)\boldsymbol{n} - \frac{1}{\mathrm{Re}}\left(\frac{\partial u_x}{\partial n}v_x + \frac{\partial u_y}{\partial n}v_y\right)\right\} d\gamma = 0$$

Reminder

Stokes equations

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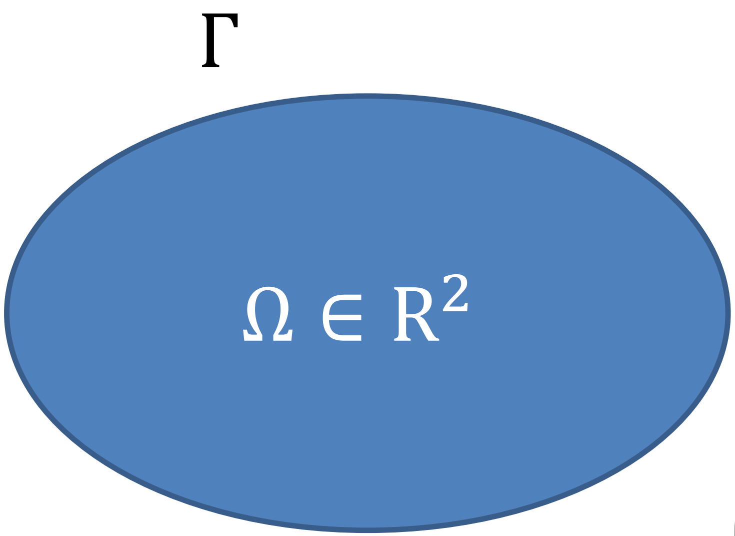
- Describe the following equation in weak form.



$$\nabla p - \frac{1}{\mathrm{Re}} \Delta \boldsymbol{u} = 0 \text{ in } \Omega$$

$$\boldsymbol{u} = \boldsymbol{0} \text{ on } \Gamma$$

$$\int \begin{bmatrix} -(\nabla \cdot \boldsymbol{v})p \\ +\frac{1}{\mathrm{Re}}\left(\frac{\partial u_x}{\partial x}\frac{\partial v_x}{\partial x} + \frac{\partial u_x}{\partial y}\frac{\partial v_x}{\partial y}\right) \\ +\frac{1}{\mathrm{Re}}\left(\frac{\partial u_y}{\partial x}\frac{\partial v_y}{\partial x} + \frac{\partial u_y}{\partial y}\frac{\partial v_y}{\partial y}\right) \\ -(\nabla \cdot \boldsymbol{u})q \end{bmatrix} d\Omega = 0$$

$$\frac{\boldsymbol{u}^{n+1}-\boldsymbol{u}^{n}}{\Delta t}+(\boldsymbol{u}^{\boldsymbol{n}}\cdot\nabla)\boldsymbol{u}^{n}+\nabla p^{n+1}-\frac{1}{\mathrm{Re}}\Delta\boldsymbol{u}^{n+1}=0,\nabla\cdot\boldsymbol{u}^{n+1}=0$$

$$\Longrightarrow\int_{\Omega}\left(\frac{\boldsymbol{u}^{n+1}-\boldsymbol{u}^{n}}{\Delta t}+(\boldsymbol{u}^{\boldsymbol{n}}\cdot\nabla)\boldsymbol{u}^{n}+\nabla p^{n+1}-\frac{1}{\mathrm{Re}}\Delta\boldsymbol{u}^{n+1}\right)\cdot\boldsymbol{w}\,d\Omega-\int_{\Omega}(\nabla\cdot\boldsymbol{u}^{n+1})q\,d\Omega=0$$

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$$\Longrightarrow\int_{\Omega}\left(\frac{\boldsymbol{u}^{n+1}-\boldsymbol{u}^{n}}{\Delta t}+(\boldsymbol{u}^{\boldsymbol{n}}\cdot\nabla)\boldsymbol{u}^{n}+\nabla p^{n+1}-\frac{1}{\mathrm{Re}}\Delta\boldsymbol{u}^{n+1}\right)\cdot\boldsymbol{w}\,d\Omega-\int_{\Omega}(\nabla\cdot\boldsymbol{u}^{n+1})q\,d\Omega=0$$

$$\Longrightarrow\int_{\Omega}\left(\frac{\boldsymbol{u}^{n+1}}{\Delta t}\cdot\boldsymbol{w}-\frac{\boldsymbol{u}^{n}}{\Delta t}\cdot\boldsymbol{w}+\{(\boldsymbol{u}^{\boldsymbol{n}}\cdot\boldsymbol{\nabla})\boldsymbol{u}^{\boldsymbol{n}}\}\cdot\boldsymbol{w}-(\nabla\cdot\boldsymbol{w})p^{n+1}+\frac{1}{\mathrm{Re}}\nabla\boldsymbol{u}^{n+1}\cdot\nabla\boldsymbol{w}\right)d\Omega$$

$$+\int_{\Gamma}\left(p^{n+1}\boldsymbol{n}-\frac{1}{\mathrm{Re}}(\nabla\boldsymbol{u}^{\mathrm{n+1,T}})\boldsymbol{n}\right)\cdot\boldsymbol{w}\,d\gamma-\int_{\Omega}(\nabla\cdot\boldsymbol{u}^{n+1})q\,d\Omega=0$$

By taking integration by parts

$$\frac{\boldsymbol{u}^{n+1}-\boldsymbol{u}^{n}}{\Delta t}+(\boldsymbol{u}^{\boldsymbol{n}}\cdot\nabla)\boldsymbol{u}^{n}+\nabla p^{n+1}-\frac{1}{\mathrm{Re}}\Delta\boldsymbol{u}^{n+1}=0,\nabla\cdot\boldsymbol{u}^{n+1}=0$$

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$$+\int_{\Gamma}\left(p^{n+1}\boldsymbol{n}-\frac{1}{\mathrm{Re}}(\nabla\boldsymbol{u}^{\mathrm{n+1,T}})\boldsymbol{n}\right)\cdot\boldsymbol{w}d\gamma-\int_{\Omega}(\nabla\cdot\boldsymbol{u}^{n+1})qd\Omega=0$$

$$\Rightarrow\int_{\Omega}\left(\frac{\boldsymbol{u}^{n+1}}{\Delta t}\cdot\boldsymbol{w}-\frac{\boldsymbol{w}}{\Delta t}convect(\boldsymbol{u}^{\boldsymbol{n}},-\Delta t,u^{n})-(\nabla\cdot\boldsymbol{w})p^{n+1}+\frac{1}{\mathrm{Re}}\nabla\boldsymbol{u}^{n+1}\cdot\nabla w\right)d\Omega$$
$$+\int_{\Gamma}\left(p^{n+1}\boldsymbol{n}-\frac{1}{\mathrm{Re}}(\nabla\boldsymbol{u}^{\mathrm{n+1,T}})\boldsymbol{n}\right)\cdot\boldsymbol{w}d\gamma-\int_{\Omega}(\nabla\cdot\boldsymbol{u}^{n+1})qd\Omega=0$$

By using “convect” command

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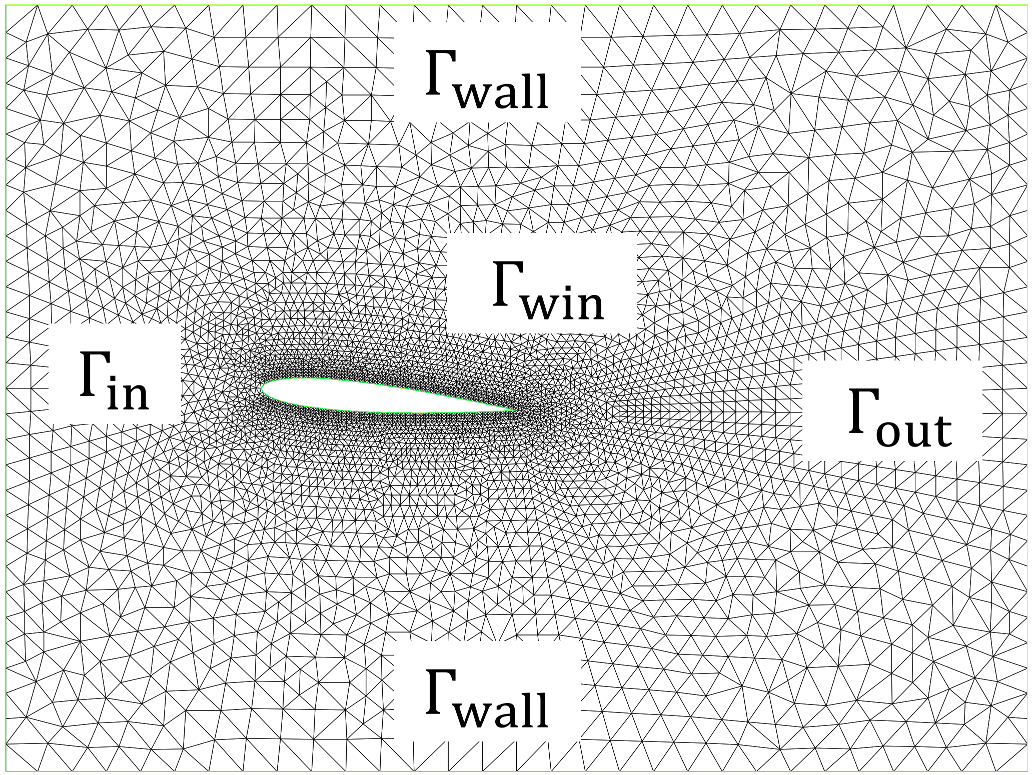

$$\frac{\partial \boldsymbol{u}}{\partial t} + (\boldsymbol{u}\cdot\nabla)\boldsymbol{u} = -\nabla p + \frac{1}{\mathrm{Re}}\Delta\boldsymbol{u} \text{ in } \Omega$$

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Velocity; $\boldsymbol{u} = (u, v)$, Pressure p

$$\frac{\boldsymbol{u}^{n+1}-\boldsymbol{u}^{n}}{\Delta t}+(\boldsymbol{u}^{\boldsymbol{n}}\cdot\nabla)\boldsymbol{u}^{n}+\nabla p^{n+1}-\frac{1}{\mathrm{Re}}\Delta\boldsymbol{u}^{n+1}=0,\nabla\cdot\boldsymbol{u}^{n+1}=0$$

$$\Rightarrow\int_{\Omega}\left(\frac{\boldsymbol{u}^{n+1}-\boldsymbol{u}^{n}}{\Delta t}+(\boldsymbol{u}^{\boldsymbol{n}}\cdot\nabla)\boldsymbol{u}^{n}+\nabla p^{n+1}-\frac{1}{\mathrm{Re}}\Delta\boldsymbol{u}^{n+1}\right)\cdot\boldsymbol{w}\,d\Omega-\int_{\Omega}(\nabla\cdot\boldsymbol{u}^{n+1})q\,d\Omega=0$$

$$\Rightarrow\int_{\Omega}\left(\frac{\boldsymbol{u}^{n+1}}{\Delta t}\cdot\boldsymbol{w}-\frac{\boldsymbol{u}^{n}}{\Delta t}\cdot\boldsymbol{w}+\{(\boldsymbol{u}^{\boldsymbol{n}}\cdot\boldsymbol{\nabla})\boldsymbol{u}^{\boldsymbol{n}}\}\cdot\boldsymbol{w}-(\nabla\cdot\boldsymbol{w})p^{n+1}+\frac{1}{\mathrm{Re}}\nabla\boldsymbol{u}^{n+1}\cdot\nabla\boldsymbol{w}\right)d\Omega$$

$$+\int_{\Gamma}\left(p^{n+1}\boldsymbol{n}-\frac{1}{\mathrm{Re}}(\nabla\boldsymbol{u}^{\mathrm{n+1,T}})\boldsymbol{n}\right)\cdot\boldsymbol{w}\,d\gamma-\int_{\Omega}(\nabla\cdot\boldsymbol{u}^{n+1})q\,d\Omega=0$$

$$\Rightarrow\int_{\Omega}\left(\frac{\boldsymbol{u}^{n+1}}{\Delta t}\cdot\boldsymbol{w}-\frac{\boldsymbol{w}}{\Delta t}convect(\boldsymbol{u}^{\boldsymbol{n}},-\Delta t,u^{n})-(\nabla\cdot\boldsymbol{w})p^{n+1}+\frac{1}{\mathrm{Re}}\nabla\boldsymbol{u}^{n+1}\cdot\nabla w\right)d\Omega$$

$$+\int_{\Gamma_{\mathrm{in}}}\left(p^{n+1}\boldsymbol{n}-\frac{1}{\mathrm{Re}}(\nabla\boldsymbol{u}^{n+1,\mathrm{T}})\boldsymbol{n}\right)\cdot\boldsymbol{w}\,d\gamma+\int_{\Gamma_{\mathrm{wall}}\cup\Gamma_{\mathrm{win}}}\left(p^{n+1}\boldsymbol{n}-\frac{1}{\mathrm{Re}}(\nabla\boldsymbol{u}^{n+1,\mathrm{T}})\boldsymbol{n}\right)\cdot\boldsymbol{w}\,d\gamma$$

$$+\int_{\Gamma_{\mathrm{out}}}\left(p^{n+1}\boldsymbol{n}-\frac{1}{\mathrm{Re}}(\nabla\boldsymbol{u}^{n+1,\mathrm{T}})\boldsymbol{n}\right)\cdot\boldsymbol{w}\,d\gamma-\int_{\Omega}(\nabla\cdot\boldsymbol{u}^{n+1})q\,d\Omega=0$$

Boundary Conditions

$$\frac{\boldsymbol{u}^{n+1}-\boldsymbol{u}^{n}}{\Delta t}+(\boldsymbol{u}^{\boldsymbol{n}}\cdot\nabla)\boldsymbol{u}^{n}+\nabla p^{n+1}-\frac{1}{\mathrm{Re}}\Delta\boldsymbol{u}^{n+1}=0,\nabla\cdot\boldsymbol{u}^{n+1}=0$$

$$\Longrightarrow\int_{\Omega}\left(\frac{\boldsymbol{u}^{n+1}-\boldsymbol{u}^{n}}{\Delta t}+(\boldsymbol{u}^{\boldsymbol{n}}\cdot\nabla)\boldsymbol{u}^{n}+\nabla p^{n+1}-\frac{1}{\mathrm{Re}}\Delta\boldsymbol{u}^{n+1}\right)\cdot\boldsymbol{w}d\Omega-\int_{\Omega}(\nabla\cdot\boldsymbol{u}^{n+1})qd\Omega=0$$

$$\Longrightarrow\int_{\Omega}\left(\frac{\boldsymbol{u}^{n+1}}{\Delta t}\cdot\boldsymbol{w}-\frac{\boldsymbol{u}^{n}}{\Delta t}\cdot\boldsymbol{w}+\{(\boldsymbol{u}^{\boldsymbol{n}}\cdot\boldsymbol{\nabla})\boldsymbol{u}^{\boldsymbol{n}}\}\cdot\boldsymbol{w}-(\nabla\cdot\boldsymbol{w})p^{n+1}+\frac{1}{\mathrm{Re}}\nabla\boldsymbol{u}^{n+1}\cdot\nabla\boldsymbol{w}\right)d\Omega$$

$$+\int_{\Gamma}\left(p^{n+1}\boldsymbol{n}-\frac{1}{\mathrm{Re}}(\nabla\boldsymbol{u}^{\mathrm{n+1,T}})\boldsymbol{n}\right)\cdot\boldsymbol{w}d\gamma-\int_{\Omega}(\nabla\cdot\boldsymbol{u}^{n+1})qd\Omega=0$$

$$\Longrightarrow\int_{\Omega}\left(\frac{\boldsymbol{u}^{n+1}}{\Delta t}\cdot\boldsymbol{w}-\frac{\boldsymbol{w}}{\Delta t}convect(\boldsymbol{u}^{\boldsymbol{n}},-\Delta t,u^{n})-(\nabla\cdot\boldsymbol{w})p^{n+1}+\frac{1}{\mathrm{Re}}\nabla\boldsymbol{u}^{n+1}\cdot\nabla w\right)d\Omega$$

$$+\int_{\Gamma_{\mathrm{in}}}\left(p^{n+1}\boldsymbol{n}-\frac{1}{\mathrm{Re}}(\nabla\boldsymbol{u}^{n+1,\mathrm{T}})\boldsymbol{n}\right)\cdot\boldsymbol{w}d\gamma+\int_{\Gamma_{\mathrm{wall}}\cup\Gamma_{\mathrm{win}}}\left(p^{n+1}\boldsymbol{n}-\frac{1}{\mathrm{Re}}(\nabla\boldsymbol{u}^{n+1,\mathrm{T}})\boldsymbol{n}\right)\cdot\boldsymbol{w}d\gamma$$

$$+\int_{\Gamma_{\mathrm{out}}}\left(p^{n+1}\boldsymbol{n}-\frac{1}{\mathrm{Re}}(\nabla\boldsymbol{u}^{n+1,\mathrm{T}})\boldsymbol{n}\right)\cdot\boldsymbol{w}d\gamma-\int_{\Omega}(\nabla\cdot\boldsymbol{u}^{n+1})qd\Omega=0$$

$$\Longrightarrow\int_{\Omega}\left(\frac{\boldsymbol{u}^{n+1}}{\Delta t}\cdot\boldsymbol{w}-\frac{\boldsymbol{w}}{\Delta t}convect(\boldsymbol{u}^{\boldsymbol{n}},-\Delta t,u^{n})-(\nabla\cdot\boldsymbol{w})p^{n+1}+\frac{1}{\mathrm{Re}}\nabla\boldsymbol{u}^{n+1}\cdot\nabla\boldsymbol{w}\right)d\Omega$$

$$-\int_{\Omega}(\nabla\cdot\boldsymbol{u}^{n+1})qd\Omega=0$$

Problem8
(The same domain as Problem1)

$$\frac{\partial \boldsymbol{u}}{\partial t} + (\boldsymbol{u} \cdot \nabla)\boldsymbol{u} = -\nabla p + \frac{1}{\mathrm{Re}} \Delta \boldsymbol{u} \text{in } \Omega$$

$$\nabla \cdot \boldsymbol{u} = 0 \text{ in } \Omega$$

Parameters: $\mathrm{Re} = 10, \Delta t = 0.05$

Initial condition: $u = -y(y-5), v = 0$

Boundary conditions:

$$u_1 = 0, u_2 = 0 \text{ on } \Gamma_{\mathrm{top}}$$

$$u_1 = 0, u_2 = 0, \text{on } \Gamma_{\mathrm{bottom}}$$

$$u_1 = -y(y-5), u_2 = 0 \text{ on} \Gamma_{\mathrm{in}}$$

$$\left(p\boldsymbol{n} - \frac{1}{\mathrm{Re}}(\nabla \boldsymbol{u}^{\mathrm{T}})\boldsymbol{n}\right) = \boldsymbol{0} \quad \text{on } \Gamma_{\mathrm{out}}$$

$$u_1 = 0, u_2 = 0 \text{ on } \Gamma_{\mathrm{circle}}$$

Problem9
(The same domain as Problem1)

$$\frac{\partial \boldsymbol{u}}{\partial t} + (\boldsymbol{u} \cdot \nabla)\boldsymbol{u} = -\nabla p + \frac{1}{\mathrm{Re}} \Delta \boldsymbol{u}, \nabla \cdot \boldsymbol{u} = 0 \text{ in } \Omega$$

$$\frac{\partial f}{\partial t} + (\boldsymbol{u} \cdot \nabla) f = \Delta f \text{ in } \Omega$$

Parameters: $\mathrm{Re} = 10, \Delta t = 0.05$

Initial condition:

$u = -y(y-5), v = 0, f = \exp\{-10(x-1)^2 + (y-1.5)^2\}$

Boundary conditions:

$$u_1 = 0, u_2 = 0, f = 0 \text{ on } \Gamma_{\text{top}}$$

$$u_1 = 0, u_2 = 0, f = 0 \text{ on } \Gamma_{\text{bottom}}$$

$$u_1 = -y(y-5), u_2 = 0, \frac{\partial f}{\partial n} = 0 \text{ on} \Gamma_{\text{in}}$$

$$\left(p\boldsymbol{n} - \frac{1}{\mathrm{Re}}(\nabla \boldsymbol{u}^{\mathrm{T}})\boldsymbol{n}\right) = \mathbf{0}, \frac{\partial f}{\partial n} = 0 \text{ on } \Gamma_{\text{out}}$$

$$u_1 = 0, u_2 = 0, f = 0 \text{ on } \Gamma_{\text{circle}}$$

Problem 10
(The same domain as “AOA_aerofoil.edp”)

- Compute the source code for the following parameters
  ソースコードを以下のパラメーターで計算せよ
  - AOA: 0, 5, 10, 15 degree
  - Re: 100, 200, 400, 600, 800, 1000

- Evaluate the Drag and the Lift coefficient
  抗力，揚力を評価せよ

- Compare the Drag-Lift coefficient for various AOA and Re.
  様々なパラメーターにおいて，揚抗比を比較せよ