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Program

- 10:10–11:00, Masahito Hayashi
  Discrimination of two channels by adaptive methods and its application to quantum system
  Abstract: The optimal exponential error rate for adaptive discrimination of two channels is discussed. In this problem, adaptive choice of input signal is allowed. This problem is discussed in various settings. It is proved that adaptive choice does not improve the exponential error rate in these settings. These results are applied to quantum state discrimination.

- 11:10–12:00, Dénes Petz
  From f-divergence to quasi-entropy

- 13:30–14:10, Fernando Brandao
  A generalization of quantum Stein's Lemma
  Abstract: Given several copies of a quantum system which is known to be described either by the state ρ or σ, called null and alternative hypotheses, what is the best measurement we can perform to learn the identity of the state at hand? A possible setting - called asymmetric hypothesis testing - considers the case in which we want to minimize, to the extreme, the probability of error of

mistakenly identifying ρ when σ is the state of the system, while only requiring that the probability that σ is identified in the place of ρ goes to zero, at any rate, when the number of copies goes to infinity. Quantum Stein's Lemma gives the rate of the exponential decaying probability of error for the optimal measurement.

In this talk we present a generalization of quantum Stein's Lemma to the situation in which the alternative hypothesis is not a single i.i.d. quantum state, but actually a family of states, which can moreover be non-i.i.d.. We consider sets of states which satisfy some natural properties, the most important being the closedness under permutations of the copies. Employing the recently established exponential quantum de Finetti theorem, we determine the rate function of the probability of the error in a very similar fashion to quantum Stein's Lemma, in terms of the relative entropy.

Although this result is not directly concerned with entanglement theory, it has interesting applications to it, two of which are discussed in the talk. First it gives an operational meaning to an entanglement measure known as regularized relative entropy of entanglement. Second, it shows that this measure is faithful, being strictly positive on every entangled state. This implies, in particular, that whenever a multipartite state can be converted into another entangled state asymptotically by local operations and classical communication, the rate of conversion must be non-zero. Therefore, the operational definition of multipartite entanglement is equivalent to its mathematical definition.

- 14:20–15:00, Attila Magyar
  From quantum state tomography to quantum channel estimation
- 15:10–15:50, Hiroshi Imai
  Fourier Analytic Method in Phase Estimation Problem
  Abstract: We analyze the limiting distribution in the phase estimation systematically, and show that the limiting distribution is expressed by Fourier transform of a square integrable function on the closed interval $[-1, 1]$, which approximately gives the input state. Using this relation, we study the relation between the variance of the limiting distribution and its tail probability. As our result, we prove that the protocol minimizing the asymptotic variance does not minimize the tail probability. We show that the protocol minimize the tail probability out of a given interval depends on the width of interval. Such an optimal protocol is given by a prolate spheroidal wave function which often appears in wavelet or time-limited Fourier analysis. We also study optimal interval estimation. We provide the optimal estimation protocol in the sense that the width of confidence interval which assures given confidence coefficient is minimized.

  This work is a joint work with Masahito Hayashi.
- 16:00–16:40, Tomohiro Ogawa
  On Asymptotic Sufficiency of Classical and Quantum Channel
- 16:50–17:30, Hiormichi Ohno
  Quasi-orthogonal algebras of matrix algebra