On the fine asymptotic behavior of the solutions for the linearized Gel'fand problem

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We consider the Gel'fand problem $-\Delta u = \lambda e^u$ and its linearized problem $-\Delta v = \lambda e^u v$ in a two-dimensional bounded domain Ω under the Dirichlet boundary condition, where λ is a positive small parameter. The Gel'fand problem and its variants are known to appear in a wide variety of areas of mathematical science such as conformal deformations of surfaces, equilibrium states of vortices, stationary states of chemotactic motions, and so forth. It is now well understood that a sequence of solutions $\{u_n\}$ for the Gel'fand problem may blow-up on some finite number of points (*blow-up points*) as $\lambda_n \downarrow 0$. In this talk, we are interested in the effect of the blowing-up behavior of u_n on the *non*-existence of non-trivial solutions for the linearized problem (*non-degeneracy* of u_n) and give a sufficient condition for the non-degeneracy in terms of the blow-up points. We prove this by a contradiction argument, supposing that there exists a sequence of non-trivial solutions $\{v_n\}$ of the linearized problem as $\lambda_n \downarrow 0$. The main ingredient of the proof is fine observation of the asymptotic behavior of $\{v_n\}$ as $\lambda_n \downarrow 0$, which is established by the new usage of the generalized Rellich identity. We also would like to discuss some other applications of the identity in the talk.

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