Existence and nonexistence of solutions for the heat equation with a superlinear source term

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We consider the heat equation with a superlinear source term:

$$\begin{cases} \partial_t u = \Delta u + f(u) & \text{in} \quad \mathbb{R}^N \times (0,T), \\ u(x,0) = u_0(x) \geq 0 & \text{in} \quad \mathbb{R}^N, \end{cases} \tag{P}$$

where $\partial_t = \partial/\partial t$, $N \geq 1$, $T > 0$, u_0 is a nonnegative initial function and f is a positive monotonically increasing function in $(0,\infty)$ with superlinear growth. We consider the case $u_0 \notin L^\infty(\mathbb{R}^N)$ and investigate local in time existence and nonexistence of solutions for problem (P) without any concrete assumption on the growth rate of f. In particular, we reveal the threshold integrability of u_0 to classify existence and nonexistence of solutions for problem (P). This is a joint work with Professor Yohei Fujishima(Shizuoka University).

References

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