Heat equation with a nonlinear boundary condition and growing initial data

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Abstract

We discuss the solvability and the comparison principle for the heat equation with a nonlinear boundary condition

$$\begin{cases} \partial_t u = \Delta u, & x \in \Omega,\ t > 0, \\ \nabla u \cdot \nu(x) = u^p, & x \in \partial\Omega,\ t > 0, \\ u(x,0) = \varphi(x) \geq 0, & x \in \Omega, \end{cases}$$

where $N \geq 1$, $p > 1$, Ω is a smooth domain in $\mathbf{R}^N$ and $\varphi(x) = O(e^{-\lambda d(x)^2})$ as $d(x) \to \infty$ for some $\lambda \geq 0$. Here $d(x) = \operatorname{dist}(x, \partial\Omega)$. Furthermore, we obtain the lower estimates of the blow-up time of solutions with large initial data by use of the behavior of the initial data near the boundary $\partial\Omega$. This talk is based on a joint work with professor Ishige (Tohoku University).

References

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