Hadamard matrices of order $3m+1$ admitting a group of order m

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Abstract. In this talk we consider a Hadamard 2-design $\mathcal{D} = (\mathcal{P}, \mathcal{B})$ of order $3m$ with an automorphism group G of order m acting semiregularly on both $\mathcal{P}$ and $\mathcal{B}$. The Hadamard matrix H corresponding to the design $\mathcal{D}$ has the following form up to equivalence:

$$H = \begin{pmatrix} H_{0,0} & H_{0,1} & H_{0,2} & J^T \\ H_{1,0} & H_{1,1} & H_{1,2} & J^T \\ H_{2,0} & H_{2,1} & H_{2,2} & J^T \\ J & J & J & 1 \end{pmatrix}, \qquad (*)$$

where each $H_{i,j} = [{h_{\alpha\beta^{-1}}}^{(i,j)}]_{\alpha,\beta\in G}$ is a $m \times m$ square matrix on $\{\pm 1\}$ and $J = \underbrace{(1, \cdots, 1)}_{m}$. Then, we show that if a prime p divides the square free part $(3m+1)^*$ of $3m+1$, $p \not\equiv 2 \pmod 3$. In fact we can obtain this result only under the condition that

$$\begin{pmatrix} H_{0,0} & J^T \\ H_{1,0} & J^T \\ H_{2,0} & J^T \\ J & 1 \end{pmatrix}^T$$

is a difference matrix on $\{\pm 1\}$.

The constructions of sequences of Hadamard matrices H's with the form $(*)$ by Paley, Singer, Hall Jr are known. We can define a *quasi-Hadamard difference set*, which is a generalization of the Hadamard difference set, corresponding to the above Hadamard matrix H. (We can consider also a more general form as H.) The representation quasi-Hadamard difference sets for Hadamard matrices H's with the form $(*)$ is useful, when we seek Hadamard matrices H's. Finally we seek the other small Hadamard matrices H's using a computer. (This is a joint work with Y. Hiramine and J. Seberry.)