ON REPRESENTATIONS ON A MODULE OVER A RING AS GENERALIZATION OF REPRESENTATIONS ON A VECTOR SPACE: A POSSIBLE USE IN CODING THEORY

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Abstract

In the coding theory, there exists a need to work on more general structure namely working on a linear code over module over a ring. In linear code now there a challenge to work on module over a ring because it is not enough just work on a vector space over a field. So from theoretical point of view, it is very reasonable to contribute on this need by generalization of ring representations and group representations on a vector space to ring representations and group representations on a module over a ring.

Let R be a commutative ring with identity and M be a free R-module then we always have a representation of R, namely ring homomorphism $\mu\colon R \to End_R(M)$, with $\mu(r) := \mu_r\colon M \to M$ and $\mu_r(m) = rm$ for all $r \in R$ and $m \in M$. In this talk, we will present some properties of representations of ring R on R-modules, based on some notions related to ring representations of R on vector spaces, such as admissible submodules, equivalence of two representations, decomposable representations and completely reducible representations. It will be shown that if M, N are two free R-modules then two representations $\mu\colon R \to End_R(M)$ and $\varphi\colon R \to End_R(N)$ are equivalent if and only if there is a module homomorphism $T\colon M \to N$. If R is a principle ideal domain, then it will be shown that every submodule of M is an admissible submodule, every representations of ring R on free R-module is decomposable, and a representation of R on M is completely reducible if and only if M is semisimple.

It is investigated a class of rings that generalizes the class of simple Artinian rings. To develop these rings we need the following bi-module structures: If M is an R-module and the ring of homomorphism on $H = End_R(M)$, then M is a left H-module under the addition already present on M and under the H-action on M given by $h \cdot x = h(x)$ for h in H and x in M. From this we can see that M is an (H, R)-bimodule. Using the ring $H = End_R(M)$, and write f in $H = End_R(M)$ on the right of the argument x in M opposite that of the ring action of H on M, then M is an $(H, End_H(M)$-bimodule. An ideal P of ring R is said to be right primitive in R if P is the largest ideal contained in some maximal right ideal of R, and R is said to be a right primitive ring if zero is a right primitive ideal. Left primitive ideals and left primitive rings are similarly defined, A ring that is left and right primitive is said to be primitive ring. With these bimodule structures in mind we will prove the density properties in relation to the primitiveness.