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Small Eigenvalues on p-forms on the Even Dimensional Spheres and the Gap Problem

Junya Takahashi

There are many studies on the eigenvalues of the Laplacian acting on functions on closed Riemannian manifolds. In the case of the Laplacian acting on differential forms, we have less knowledge of the eigenvalues. Lott (1999) established a convergence theorem of the eigenvalues on p-forms for a family of closed Riemannian manifolds with bounded sectional curvature and bounded diameter. One of the interesting phenomena is to exist small eigenvalues on p-forms which tend to 0. However, except for the special cases, we do not know results on small eigenvalues in the case of the sectional curvature bounded only below. So we study special examples in this case. T. Yamaguchi constructed the collapsing such that (S^{2n}, g_ε) $(n \geq 2)$ converges to the spherical suspension of the complex projective space with non-negative sectional curvature as $\varepsilon \to 0$. We denote by $\lambda_1^{(p)}(M, g)$ the first positive eigenvalue of the Laplacian acting on p-forms on (M, g).

Theorem 1. *We obtain*

$$\lambda_1^{(0)}(S^{2n}, g_\varepsilon) \quad \geq \quad C \text{ and } \lambda_1^{(p)}(S^{2n}, g_\varepsilon) \to 0,$$

as $\varepsilon \to 0$, for $1 \leq p \leq 2n-1$ except for p being n and odd. Here, C is a positive constant independent of ε.

Similarly, we obtain the collapsing (S^{4n}, g_ε) such that $\lambda_1^{(p)}(S^{4n}, g_\varepsilon) \to 0$ as $\varepsilon \to 0$ for some p. Recently, Lott (2002) proved the existence of small eigenvalues on p-forms in the more general case. Especially, it follows that $\lambda_1^{(p)}(S^{2n}, g_\varepsilon) \to 0$ for all $1 \leq p \leq 2n-1$.

Next, we study whether any closed manifold M admits a metric such that the gap between $\lambda_1^{(p)}(M, g)$ and $\lambda_1^{(0)}(M, g)$ exists. Then, we obtain the following.

Theorem 2. *Let M^m be any $m \geq 4$ dimensional connected oriented closed manifold. Then, for $2 \leq p \leq m-2$, there exist three metrics g_i $(i = 1, 2, 3)$ on M such that*

$$
\begin{aligned}
\lambda_1^{(p)}(M, g_1) &> \lambda_1^{(0)}(M, g_1),\\
\lambda_1^{(p)}(M, g_2) &< \lambda_1^{(0)}(M, g_2)\\
&\textit{and}\\
\lambda_1^{(p)}(M, g_3) &= \lambda_1^{(0)}(M, g_3).
\end{aligned}
$$

We see a similar result for $p = 1$.

Finally, when we impose some geometric conditions on a Riemannian manifold, we consider the gap between $\lambda_1^{(p)}(M, g)$ and $\lambda_1^{(0)}(M, g)$. If $\lambda_1^{(1)}(M, g) < \lambda_1^{(0)}(M, g)$ holds for a connected closed Einstein manifold (M, g) with positive Ricci curvature, then the identity map is weakly stable as a harmonic map. If (M, g) has a non-trivial parallel p-form, then it follows that $\lambda_1^{(p)}(M, g) \leq \lambda_1^{(0)}(M, g)$.

References

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On the Finiteness Conditions for Convex Hulls of Self-Similar Sets

A. V. Tetenov

Let $S = \{\varphi_1, \dots, \varphi_m\}$ be a system of contraction similitudes of $\mathbf{R}^n$ (written in a form $\varphi_i(x) = r_i \cdot Q_i(x - x_i) + x_i$, (where $0 < r_i < 1$, Q_i is an orthogonal transformation of $\mathbf{R}^n$, and x_i is a fixed point of φ_i) .

A compact nonempty set K is called an *invariant set* of the system S [1] if it satisfies $K = \bigcup \varphi_i(K)$. Let $\tilde{K}$ be the convex hull of the set K and F be the set of extreme points of $\tilde{K}$.

In our recent work [2] it was shown that a convex hull of a self-similar set in $\mathbb{R}^n$ may have infinite set of sides and uncountable set of extreme points. It was also shown in [2] that if a system $S = \{\varphi_1, \dots, \varphi_m\}$ of contracting

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