

# Markov chains, graph spectra, and some static/dynamic scaling limits

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I will talk about how I began to get interested in spectra of graphs and then was led to beautiful collaboration with N. Obata. Furthermore I will combine them with recent developments in probability models concerning Young diagrams. Key words are cut-off phenomenon, association scheme, quantum probability, free probability, and asymptotic representation theory.

**Three Lectures on the Terwilliger algebra  
of  
a (P and Q)-polynomial association scheme**

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**First Lecture:** (P and Q)-polynomial association schemes and the Leonard theorem

1. The definition of a (P and Q)-polynomial association scheme
2. Examples (Bannai's list)
3. The Leonard theorem
4. The Terwilliger algebra and its principal module

**Second Lecture:** L-pairs, TD-pairs and the TD-algebra

1. L-pairs and TD-pairs
2. The TD-relations and the TD-algebra
3. The weight space decomposition and the augmented TD-algebra
4. TD-pairs and the quantum affine algebra  $U_q(\widehat{\mathfrak{sl}}_2)$

**Third Lecture:** Toward the classification of (P and Q)-polynomial schemes

1. The classification of TD-pairs
2. The present status of the classification of (P and Q)-polynomial schemes
3. Irreducible T-modules of endpoint 1, 2

# SOME RAMSEY NUMBERS AND RAMSEY $(mK_2, H)$ -MINIMAL GRAPHS

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In the first talk, we shall give a survey on the finding of Ramsey numbers  $R(G, H)$  if one of  $G$  and  $H$  is a wheel. We also discuss the Ramsey numbers  $R(G, H)$  if either  $G$  or  $H$  is a union of graphs.

In the second talk, we will derive the necessary and sufficient conditions of Ramsey  $(mK_2, H)$ -minimal graphs. We will also determine all Ramsey  $(mK_2, H)$ -minimal graphs for some particular graphs  $H$ . Some construction methods of such Ramsey minimal graphs from the existing (smaller) Ramsey minimal graphs are also presented.

# The locating-chromatic number of trees with maximum degree 3 or 4

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## Abstract

A  $k$ -coloring of  $G$  is a function  $c : V(G) \rightarrow \{1, 2, \dots, k\}$  where  $c(u) \neq c(v)$  for two adjacent vertices  $u$  and  $v$  in  $G$  and  $k$  is a positive integer. The partition  $\pi = \{C_1, C_2, \dots, C_k\}$  is induced by the  $k$ -coloring  $c$  of the vertices of  $G$ . The color code of vertex  $v$  is  $c_\pi(v) = (d(v, C_1), d(v, C_2), \dots, d(v, C_k))$  where  $d(v, C_i) = \min\{d(v, x) | x \in C_i\}$  for  $1 \leq i \leq k$ . If all distinct vertices of  $G$  have distinct color codes, then  $c$  is called a locating  $k$ -coloring of  $G$ . The locating chromatic number of  $G$ , denoted by  $\chi_L(G)$  is the least integer  $k$  such that  $G$  has a locating  $k$ -coloring.

In this talk we will discuss the locating-chromatic number of trees embedded in 2-dimensional grid and binary trees. This is an attempt to answer an open problem of determining the locating-chromatic number of trees with maximum degree 3 or 4

## Resolvable Steiner designs and maximal arcs in projective planes

Vladimir D. Tonchev, Michigan Technological University

Let  $D = \{X, \mathcal{B}\}$  be a Steiner  $2-(v, k, 1)$  design with point set  $X$ , collection of blocks  $\mathcal{B}$ , and let  $v$  be a multiple of  $k$ ,  $v = nk$ . A *parallel class* is a set of  $v/k = n$  pairwise disjoint blocks that partition  $X$ , and a *resolution* is a partition  $R$  of  $\mathcal{B}$  into  $r = (v - 1)/(k - 1)$  disjoint parallel classes. A design is *resolvable* if it admits a resolution. Two resolutions  $R_1, R_2$ ,

$$R_1 = P_1^{(1)} \cup P_2^{(1)} \cup \dots \cup P_r^{(1)}, \quad R_2 = P_1^{(2)} \cup P_2^{(2)} \cup \dots \cup P_r^{(2)}$$

are called *compatible* [1] if they share one parallel class,  $P_i^{(1)} = P_j^{(2)}$ , and  $|P_{i'}^{(1)} \cap P_{j'}^{(2)}| \leq 1$  for  $(i', j') \neq (i, j)$ .

A maximal  $(q(k - 1) + k, k)$ -arc in a finite projective plane of order  $q = sk$  is a set  $A$  of  $q(k - 1) + k$  points such that every line is either disjoint from  $A$ , or meets  $A$  in exactly  $k$  points.

An upper bound on the maximum number of mutually compatible resolutions of a resolvable  $2-(nk, k, 1)$  design  $D$  was proved in [1]. The bound is attainable if and only if  $D$  is embeddable as a maximal  $(kq - q + k, k)$ -arc in a projective plane of order  $q = (v - k)/(k - 1)$ .

The maximal sets of mutually compatible resolutions of  $2-(52, 4, 1)$  designs associated with known and newly found maximal  $(52, 4)$ -arcs in projective planes of order 16 were computed recently in [2]. It was shown that some  $2-(52, 4, 1)$  designs can be embedded as maximal arcs in nonisomorphic planes. This phenomenon establishes new links between the known planes of order 16, and motivates the problem of completing the classification of maximal  $(52, 4)$ -arcs, initiated in [3].

## References

- [1] Vladimir D. Tonchev, On resolvable Steiner 2-designs and maximal arcs in projective planes, *Designs, Codes and Cryptography* **84**, No. 1 - 2 (2017), 165 - 172.
- [2] M. Gezek, T. Wagner and V. D. Tonchev, Maximal arcs in projective planes of order 16 and related designs, *Advances in Geometry*, to appear.

- [3] N. Hamilton, S. Stoichev, and V. D. Tonchev, Maximal arcs and disjoint maximal arcs in projective planes of order 16, *J. Geometry* **67** (2000), 117-126.

## Counting Steiner triple systems of given 2-rank and 3-rank

Vladimir D. Tonchev,  
Michigan Technological University

This lecture is based on joint work with Dieter Jungnickel [3], [4].

By a famous result of Doyen, Hubaut and Vandensavel [2], the 2-rank of the incidence matrix of a Steiner triple system on  $2^n - 1$  points is at least  $2^n - 1 - n$ , and equality holds only for the classical design of points and lines in the binary projective geometry  $PG(n - 1, 2)$ . It follows from results of Assmus [1] that, given any integer  $t$  with  $1 \leq t \leq n - 1$ , there is a binary linear code  $C_{n,t}$  of length  $2^n - 1$  and dimension  $2^n - 1 - n + t$  that contains representatives of all isomorphism classes of  $STS(2^n - 1)$  of 2-rank at most  $2^n - 1 - n + t$ . Using a mixture of coding theoretic, geometric, design theoretic and combinatorial arguments, we prove a general formula for the number of distinct  $STS(2^n - 1)$  having 2-rank at most  $2^n - 1 - n + t$  contained in this code. This generalizes previously known results, which only cover the cases  $t \leq 3$  (Tonchev [5], V. Zinoviev and D. Zinoviev [7], D. Zinoviev [6]). Finally, using our recent systematic study of the ternary linear codes of Steiner triple systems [4], we obtain analogous results for the ternary case, and a formula for the number of  $STS(3^n)$  having 3-rank at most  $3^n - 1 - n + t$ .

## References

- [1] E. F. Assmus, Jr., On 2-ranks of Steiner triple systems, *Electronic J. Combinatorics* **2** (1995), paper #R9.
- [2] J. Doyen, X. Hubaut, M. Vandensavel, Ranks of incidence matrices of Steiner triple systems, *Math. Z.* **163** (1978), 251 - 259.
- [3] D. Jungnickel and V. D. Tonchev, Counting Steiner triple systems with classical parameters and prescribed rank, arXiv: 1709.06044, 18 September 18, 2017.
- [4] D. Jungnickel and V. D. Tonchev, On Bonisoli's theorem and the block codes of Steiner triple systems, *Des. Codes Cryptogr.*, DOI 10.1007/s10623-017- 0406-9.

- [5] V. D. Tonchev, A mass formula for Steiner triple systems  $STS(2^n - 1)$  of 2-rank  $2^n - n$ , *J. Combin. Th. Ser. A* **95** (2001), 197 - 208.
- [6] D. V. Zinoviev, The number of Steiner triple systems  $S(2^m - 1, 3, 2)$  of rank  $2^m - m + 2$  over  $F_2$ , *Discr. Math.* **339** (2016), 2727 - 2736.
- [7] V. A. Zinoviev, D. V. Zinoviev, Steiner triple systems  $S(2^m - 1, 3, 2)$  of rank  $2^m - m + 1$  over  $F_2$ , *Problems of Information Transmission* **48** (2012), 102 - 126.



# Using SageMath for algebraic combinatorics, in particular for strongly regular graphs

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SageMath [1] is an open-source computer algebra system, combining systems such as GAP, Singular, PARI, etc., which are glued together by a popular mainstream programming language Python. It is well-suited for rapid implementations of various combinatorial-algebraic constructions, such as block designs, Hadamard matrices, graphs, etc.—and it includes generators for many popular constructions of such objects. In particular, for each tuple of parameters in A.E. Brouwer’s tables of strongly regular graphs [2] with up to 1300 vertices for which a construction is known, SageMath can generate an example of a graph with these parameters [3]. In our lectures we will give a quick introduction to SageMath, followed by a detailed presentation of [3].

## References

- [1] <http://www.sagemath.org/>
- [2] <https://www.win.tue.nl/~aeb/graphs/srg/srgtab.html>
- [3] <https://arxiv.org/abs/1601.00181>

# A survey on Euclidean designs and relative designs

Etsuko Bannai

Euclidean  $t$ -design was introduced by Neumaier and Seidel in 1988 as a generalization of spherical designs. Euclidean  $t$ -design is a finite set in Euclidean space. We introduce the Fisher type lower bounds for the cardinality and the concept of tight  $t$ -design. The concept of relative  $t$ -design in association schemes was introduced by Delsarte in 1977 earlier than the Definition of Euclidean designs. Instead of spheres in Euclidean space we consider shells of an association scheme. Fisher type lower bound for the cardinality and the concept of tight  $t$ -design. We survey on the known results of both tight Euclidean designs and tight relative designs on some association schemes.

# Spherical designs, complex spherical designs, and unitary designs

Eiichi Bannai

We first give a survey on these concepts, following the three basic papers: Spherical codes and designs (Delsarte-Goethals-Seidel, 1977); Complex spherical designs and codes (Roy-Suda, 2014); Unitary designs and codes (Roy-Scott, 2009). Then in particular we comment on the paper of Roy-Suda (2014), and discuss the existence and the classification problems of "good" tight complex spherical  $T$ -designs (for certain  $T$ ) coming from tight real spherical  $t$ -designs. Here, "good" means either the number of distances  $s = |A(X)|$  is small, or an association scheme is naturally attached to it. The last part of this talk is based on the ongoing joint work with Takayuki Okuda (Hiroshima University), Da Zhao (Shanghai Jiao Tong University) and Yan Zhu (Shanghai University).