2008年5月30日

$$f(J) = \left|\left\{a \mid \{j \in I \mid a \notin A_j\} = J\right\}\right|$$

$$\begin{aligned} g(J) &= \sum_{\substack{K \in 2^I \\ K \subset J}} f(K) \\ &= \Big|\bigcup_{\substack{K \in 2^I \\ K \subset J}} \left\{a \mid a \in A,\ \{j \in I \mid a \notin A_j\} = K\right\}\Big| \\ &= \left|\left\{a \mid a \in A,\ \{j \in I \mid a \notin A_j\} \subset J\right\}\right| \\ &= \Big|\bigcap_{j \in \overline{J}} A_j\Big|. \end{aligned}$$

$$\begin{aligned} \Big|A - \bigcup_{i \in I} A_i\Big| &= f(I) \\ &= \sum_J \mu(J, I) g(J) \\ &= \sum_J \mu(\overline{J}, I) g(\overline{J}) \\ &= \sum (-1)^{|I| - |\overline{J}|} \Big|\bigcap_{j \in J} A_j\Big| \\ &= \sum (-1)^{|J|} \Big|\bigcap_{j \in J} A_j\Big|. \end{aligned}$$

$$\begin{aligned} &|\{f \mid f : A \to B,\ f \text{ は全射 }\}| \\ &= \Big|B^A - \bigcup_{b \in B} F_b\Big| \\ &= \sum_{C \subset B} (-1)^{|C|} \cdot \Big|\bigcap_{b \in C} F_b\Big| \\ &= \sum_{i=0}^{n} \sum_{C \in \binom{B}{i}} (-1)^i |\{f \mid f : A \to B,\ C \cap f(A) = \varnothing\}| \\ &= \sum_{i=0}^{n} \sum_{C \in \binom{B}{i}} (-1)^i |(B - C)^A| \\ &= \sum_{i=0}^{n} \tbinom{n}{i} (-1)^i (n - i)^m. \end{aligned}$$