

2011年6月28日

$$\begin{aligned}\sum_{\substack{y \in X \\ y \leq x}} \mu(y, x) g(y) &= \sum_{\substack{y \in X \\ y \leq x}} \mu(y, x) \sum_{\substack{z \in X \\ z \leq y}} f(z) \\ &= \sum_{y \in X} \mu(y, x) \sum_{z \in X} \zeta(z, y) f(z) \\ &= \sum_{z \in X} \left( \sum_{y \in X} \zeta(z, y) \mu(y, x) \right) f(z) \\ &= \sum_{z \in X} (\zeta \mu)_{z, x} f(z) \\ &= \sum_{z \in X} \delta_{z, x} f(z) \\ &= f(x).\end{aligned}$$

$$\begin{aligned}g(J) &= \sum_{\substack{K \in 2^I \\ K \subset J}} f(K) \\ &= \left| \bigcup_{\substack{K \in 2^I \\ K \subset J}} \{a \mid a \in X, \{j \in I \mid a \notin A_j\} = K\} \right| \\ &= \left| \{a \mid a \in X, \{j \in I \mid a \notin A_j\} \subset J\} \right| \\ &= \left| \{a \mid a \in X, \bar{J} \subset \{j \in I \mid a \in A_j\}\} \right| \\ &= \left| \{a \mid a \in X, \forall j \in \bar{J}, a \in A_j\} \right| \\ &= \left| \bigcap_{j \in \bar{J}} A_j \right|.\end{aligned}$$