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Theorem 1. *If Δ is a simple system in a root system Φ, then $W = \langle s_\alpha \mid \alpha \in \Delta \rangle$.*

Theorem 2. *Let Δ be a simple system in a root system Φ. Let $\alpha_1, \ldots, \alpha_r \in \Delta$ and $w = s_1 \cdots s_r \in W(\Phi)$, where $s_i = s_{\alpha_i}$ for $1 \leq i \leq r$. If $\ell(w) < r$, then there exist i, j with $1 \leq i < j \leq r$ such that*

$$w = s_1 \cdots s_{i-1} s_{i+1} \cdots s_{j-1} s_{j+1} \cdots s_r.$$

Definition 3. Let X be a set of formal symbols, and let $F(X)$ be the free group generated by the set of involutions X. Let $R \subset F(X)$. Let N be the subgroup generated by the set

$$\{c^{-1} r^{\pm 1} c \mid c \in F(X),\ r \in R\}. \tag{1}$$

In other words, N is the set of elements of $F(X)$ expressible as a product of elements in the set (1). The set

$$F(X)/N = \{aN \mid a \in F(X)\},$$

where $aN = \{ab \mid b \in N\}$ for $a \in F(X)$, forms a group under the binary operation

$$\begin{array}{rcl} F(X)/N \times F(X)/N & \to & F(X)/N \\ (aN, bN) & \mapsto & abN \end{array}$$

and it is called the group with *presentation* $\langle X \mid R \rangle$. If a group G is isomorphic to $F(X)/N$, then G is said to have *presentation* $\langle X \mid R \rangle$.