# 11th Sendai Workshop on

# Infinite Dimensional Analysis and Quantum Probability

Venue: Graduate School of Information Sciences, Tohoku University
6F Small Lecture Room
Dates: October 26-27, 2015
Organizers: Nobuaki Obata (Tohoku University), Un Cig Ji (Chungbuk National University)
Supported by JSPS-NRF Basic Scientific Cooperation Program (2015-16) "Non-commutative stochastic analysis: New prospects of quantum white noise and quantum walk"

## Program

### October 26 (Mon)

13:30 - 14:20
*Quantum Girsanov transform as solution of implementation problem*
Un Cig Ji (Chungbuk National University)

A quantum extension of classical Girsanov transform, so called quantum Girsanov transform, is formulated as an implementation problem which is equivalent to a differential equation associated with a Wick derivation. By solving the differential equation, we obtain an explicit form of the quantum Girsanov transform. For our studies, we introduce the notion of quantum white noise derivatives in the framework of quantum white noise theory and study Wick algebras of white noise operators and Wick derivations. Then we can solve the differential equations of Wick type related to the implementation problems. This talk is based on several joint works with Nobuaki Obata.

14:30 - 15:20
*Contraction coefficients for noisy quantum channels*
Fumio Hiai (Tohoku University, Professor Emeritus)

Generalized relative entropy, monotone Riemannian metrics, geodesic distance, and trace distance are all known to decrease under the action of quantum channels. We give some new bounds on, and relationships between, the maximal contraction for these quantities.

15:30 – 16:20

*The relative weak Haagerup property and the Hilbert module weak Haagerup property for C*-algebras*

Jaeseong Heo (Hanyang University)

In this paper, we introduce the notions of the relative weak Haagerup property and the Hilbert module weak Haagerup property for a pair of C*-algebras with faithful tracial states. We prove that if $\Lambda$ is a normal subgroup of a countable discrete group $\Gamma$ and if $(A, \Gamma, \alpha)$ is a unital C*-dynamical system, the pair $\left(A \rtimes_{\alpha,r} \Gamma,\ A \rtimes_{\alpha,r} \Lambda\right)$ of reduced C*-crossed products has the relative weak Haagerup property if and only if the quotient group $\Gamma/\Lambda$ has the weak Haagerup property. Finally, we prove that the weak Haagerup property of an action $\alpha: \Gamma \curvearrowright A$ is equivalent to the Hilbert $A$-module weak Haagerup property of $A \rtimes_{\alpha,r} \Gamma$.

16:30-17:20

*Quantum completeness and Liouville property of harmonic functions*

Jun Masamune (Tohoku University)

Quantum completeness and Liouville property of harmonic functions have been studied for long time in quantum mechanics and geometric analysts rather independently. In this talk we will study their relationships in the frameworks of Riemannian geometry and infinite weighted graphs.

## October 27 (Tue)

10:00 – 10:50

*Quaternionic quantum walks*

Norio Konno (Yokohama National University)

The discrete-time quantum walk has been intensively studied for the last decade, whose coin operator is given by a unitary matrix. We extend the quantum walk to a walk determined by a unitary matrix whose component is quaternion. We call this model quaternionic quantum walk and present some properties.

11:00 – 11:50

*Similarity degree of Fourier algebras*

Hun Hee Lee (Seoul National University)

Pisier introduced the concept of similarity degree to attack the problem of Dixmier's similarity problem and Kadison's similarity problem in the same context. In this talk we will explain Pisier's similarity degree for completely contractive Banach algebras and apply to the case of Fourier algebra $A(G)$. We will show that for infinite QSIN groups (containing amenable or discrete groups) the similarity degree of the corresponding Fourier algebra is exactly 2. As a consequence we prove the following Fourier algebra version of Dixmier's similarity problem: any cb-homomorphism from $A(G)$ to $B(H)$ is similar to *-representation.

13:30-14:20

*Powers of an infinite dimensional Brownian motion associated with the product of distributions*

Kimiaki Saitô (Meijo University)

In this talk we define higher powers of an infinite dimensional Brownian motion on a space consisting of distributions without any renormalization, and give an extension of the Itô formula for the Brownian motion. Moreover we extend the Lévy and Volterra Laplacians to operators on a locally convex space taking the completion of the set of all distribution-coefficient polynomials on distributions with respect to some topology, and give a relationship between those Laplacians and the generator of the Brownian motion with realizing a divergent part in mathematics .

[1] L. Accardi, U. C. Ji and K. Saitô: The exotic (higher order Lévy) Laplacians generate the Markov processes given by distribution derivatives of white noise, Infinite Dimensional Analysis, Quantum Probability and Related Topics, Vol. 16, No. 3 (2013) 1--26.
[2] H.-H. Kuo, N. Obata and K. Saitô: Lévy Laplacian of generalized functions on a nuclear space, J. Funct. Anal. 94 (1), 74--92 (1990).
[3] K. Saitô and T. Shimada: Powers of an infinite dimensional Brownian motion associated with the product of distributions, Communications on Stochastic Analysis
Vol. 8, No. 3 (2014) 343--364.

14:30-15:20

*One-dimensional three-state quantum walks: weak limits and localization*

Hyun Jae Yoo (Hankyong National University, Korea)

We investigate one-dimensional three-state quantum walks. We find a formula for the moments of the weak limit distribution via a vacuum expectation of powers of a self-adjoint operator. We use this formula to fully characterize the localization of three-state quantum walks in one-dimension. The localization is also characterized by investing the eigenvectors of the evolution operator for the quantum walk. As a byproduct we clarify the concepts of localization differently used in the literature. We also study the continuous part of the limit distribution. For typical examples we show that the continuous part is the same kind as that of two-state quantum walks. We provide with explicit expressions for the density of the weak limits of some three-state quantum walks. This is a joint work with Chul Ki Ko and Etsuo Segawa.

15:30-16:20

*Generators of some quantum walks*

Etsuo Segawa (Tohoku University)

Some limit behaviors of quantum walks belonging to named Szegedy class, is sometimes easy to analyze due to the well-known underlying cellular automata, for example, random walk. We consider an essential structure of the time-evolution of the Szegedy class. One of our simple answers is that the time evolution is expressed by product of two different involution operators. From this, we can construct more general quantum walks without notion of graphs. Conversely, we also construct a graph from the time evolution of the Szegedy class.

16:20 – 16:50

*On Stieltjes transform for augmented Jacobi parameters of Chebyshev type*

Nobuaki Obata (Tohoku University)

During the recent study of limit distributions of continuous-time quantum walks on $\mathbf{Z}_+$ with E. Segawa, N. Konno and L. Matsuoka, we have come to a slightly new concept of augmented Jacobi parameters. I will report on the Stieltjes transforms of probability measures corresponding to augmented Jacobi parameters of Chebyshev type: $\{\zeta_j\} \cup \{\omega_n = \omega\}$.