# 1st Tohoku-Bandung Bilateral Workshop: Extremal Graph Theory, Algebraic Graph Theory And Mathematical Approach to Network Science

Dates: May 29 (Mon) - June 2 (Fri), 2017

Venue: Small Lecture Room 6F, Graduate School of Information Sciences, Tohoku University Sponsors: (1) JSPS Open Partnership Joint Projects: Extremal graph theory, algebraic graph theory and mathematical approach to network science (2017-2018)
(2) Grant-in-Aid of Scientific Research 16H03939

# Program

#### May 29 (Mon)

13:15-14:15 On Ramsey (2K<sub>2</sub>,H)-minimal graph: characterization Kristiana Wijaya (Bandung Institute of Technology)
14:30-15:30 Restricted size Ramsey number for graph of size two versus graph without isolates Denny Riama Silaban (Bandung Institute of Technology)

#### May 30 (Tue)

13:15-14:15	On Ramsey $(2K_2, H)$ -minimal graph: operation
	Kristiana Wijaya (Bandung Institute of Technology)
14:30-15:00	Hadamard matrices and a matrix approach to complementary sequences
	Pritta Etriana Putri (Tohoku University)
15:15-15:45	Complementary Ramsey numbers, graph factorizations and Ramsey graphs
	Akihiro Munemasa (Tohoku University)
16:00-16:30	On Ramsey $(P_4, P_4)$ -minimal graphs.
	Yusuke Yoshie (Tohoku University)
May 31 (Wed	) Free discussion
June 1 (Thu)	)
13:15-14:00	Asymptotic spectral distributions for Cartesian powers of strongly regular graphs
	Hajime Tanaka (Tohoku University)
14:15-14:45	Some constructions of spherical designs
	Samy Baladram (Tohoku University)
14:45-15:15	A short review on a quantum search driven by quantum walks
	Etsuo Segawa (Tohoku University)
15:30-16:00	Some topics in distance matrices of graphs
	Nobuaki Obata (Tohoku University)

June 2 (Fri) Free discussion

### Abstracts

Some constructions of spherical designs Samy Baladram

In this talk, I will explain some fundamentals and well-known constructions of spherical *t*-designs. Some new recent construction from my personal research, using a so-called ball *t*-design, will also be presented.

Complementary Ramsey numbers, graph factorizations and Ramsey graphs Akihiro Munemasa

A special case of weak Ramsey numbers which we call the complementary Ramsey number is the smallest number of vertices in a graph such that in every edge coloring, there exists a complete subgraph of a specified size whose edge colors misses one. This generalizes the classical Ramsey number for 2 colors. We show that certain graph factorizations and knowledge of Ramsey graphs can help determine complementary Ramsey numbers. This is based on joint work with Masashi Shinohara.

Some topics in distance matrices of graphs Nobuaki Obata

A graph is called of QE class if it admits a quadratic embedding in Euclidean space (or Hilbert space). By Schönberg theorem this is equivalent to that the distance matrix is conditionally negative definite. In order to decide this property the quadratic embedding constant (QEC) of a graph was introduced. In this talk graph operations preserving the property of being of QE class and estimates of QEC are discussed.

Hadamard matrices and a matrix approach to complementary sequences Pritta Etriana Putri

It is known that complementary sequences has a relation with Hadamard matrices. We give a matrix approach, rather than a sequence approach, to construct complementary sequences. Two-variable Laurent polynomials and the Lagrange identity are used in our method. This talk is based on a joint work with Akihiro Munemasa.

A short review on a quantum search driven by quantum walks Etsuo Segawa

We take a short review on a quantum search driven by quantum walks on the complete graphs for simplicity. The eigenspace of the time evolution operator of the quantum search inherited from the isotropic random walk on the graph with the Dirichlet boundary condition on the marked vertices plays important role to estimate the efficiency.

Asymptotic spectral distributions for Cartesian powers of strongly regular graphs Hajime Tanaka

Generalizing previous work of Hora on the Hamming graphs which are Cartesian powers of complete graphs, we compute various limit spectral distributions of pairs of Cartesian powers of strongly regular graphs and their complements.

On Ramsey  $(P_4, P_4)$ -minimal graphs Yusuke Yoshie

For any given two graphs G and H, the notation  $F \to (G, H)$  means that any red-blue coloring of all the edges of F will create either a red subgraph isomorphic to G or a blue subgraph isomorphic to H. A graph F is a Ramsey (G, H)-minimal graph if  $F \to (G, H)$  but  $F - \{e\} \not\rightarrow$ (G, H) for every  $e \in E(F)$ . The class of all Ramsey (G, H)-minimal graphs is denoted by R(G, H). In this paper, we prove that there is no tree in  $R(P_m, P_n)$  for  $m, n \ge 4$ . We also characterize the unique kind of unicyclic graphs in  $R(P_4, P_4)$ . In particular, we determine some infinite classes of graphs belonging to  $R(P_4, P_4)$ .

### **ON RAMSEY** $(2K_2, H)$ -MINIMAL GRAPHS

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Abstract. Ramsey graphs theory deals with regularity and coloring of graphs. There are many interesting applications of Ramsey graphs theory, such as in the fields of communications, information retrieval, and decision making. Let F, G and H be simple graphs. We write  $F \to (G, H)$  to mean that any red-blue coloring on all edges of F will contain either a red copy of G or a blue copy of H. The graph F (without isolated vertices) satisfying  $F \to (G, H)$  and  $(F - e) \not\rightarrow (G, H)$  for every  $e \in E(F)$  is called a Ramsey (G, H)-minimal graph. The set of all Ramsey (G, H)-minimal graphs is denoted by  $\mathscr{R}(G, H)$ . In this paper, we derive the sufficient and necessary condition of graphs belonging to  $\mathscr{R}(mK_2, H)$ . We give a relation between Ramsey  $(mK_2, H)$ - and  $((m - 1)K_2, H)$ -minimal graphs. Furthermore, we construct graphs in  $\mathscr{R}(2K_2, H)$ , where  $H = K_4$  and  $H = C_4$ . We show that a graph obtained from any two connected graphs in  $\mathscr{R}(2K_2, H)$  by identifying a vertex or an edge is a member of  $\mathscr{R}(2K_2, 2H)$ , where H is a complete, a cycle, a path, or a star.

Keywords: Ramsey minimal graph, edge coloring, complete graph, path graph.

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# Restricted Size Ramsey Number for Graph of Size Two versus Connected Graph

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#### Abstract

Let G and H be simple graphs. The Ramsey number for G and H is the smallest number r such that any red-blue coloring of edges of  $K_r$  contains a red subgraph G or a blue subgraph H. The size Ramsey number for G and H is the smallest number  $\hat{r}$  such that there exists a graph F with size  $\hat{r}$  satisfying the property that any red-blue coloring of edges of F contains a red subgraph G or a blue subgraph H. Additionally, if the order of F is r(G, H), then it is called the restricted size Ramsey number. In 1983, Harary and Miller started to find the (restricted) size Ramsey number for any pair of small graphs. They obtained the values for some fair af small graphs with order at most four. Faudree and Sheehan (1983) continued Harary and Miller's works and summarized the complete results on the (restricted) size Ramsey number for any pair of small graphs with order at most four. Moreover, Lortz and Mengenser (1998) gave both the size Ramsey number and the restricted size Ramsey number for any pair of small forests with order at most five.

Furthermore, for any pair of graphs G and H, both the size Ramsey number  $\hat{r}(G, H)$  and the restricted size Ramsey number  $r^*(G, H)$  are bounded above by the size of the complete graph with order is equal to the Ramsey number r(G, H), and bounded below by e(G) + e(H) - 1. Moreover, trivially,  $\hat{r}(G, H) \leq r^*(G, H)$ . When both G and H are complete graphs, both the size and restricted size Ramsey number of G and H attain the upper bound. While, when both G and H are star graphs, the size Ramsey number of G and H attains the lower bound.

In this talk we consider the restricted size Ramsey number for graph of size two versus connected graph H. We give a short survey of our results concerning to the topic. They are included the necessary and sufficient conditions for a connected graph H such that the restricted size Ramsey number  $r^*(P_3, H)$  and  $r^*(2P_2, H)$  attain the upper and lower bounds, some exact values of  $r^*(P_3, H)$  and  $r^*(2P_2, H)$  for H is a connected graph, and some exact values of  $r^*(P_3, H)$  and  $r^*(2P_2, H)$  for H is any small connected graph of order five.

**Keywords :** restricted size Ramsey number, path, matching, connected graph

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