9th Sendai Workshop

Infinite Dimensional Analysis and Quantum Probability

September 11 - 12, 2009
Graduate School of Information Sciences
Tohoku University

PROGRAM

September 11 (Fri) GSIS, Large Lecture Hall (2F)

1000-1010 Takao Nishizeki (Dean, GSIS, Tohoku University)
Opening Address

1010-1050 Marek Bożejko (University of Wrocław, Poland)
Why normal law is infinitely divisible in the free probability? Connections with generalized Brownian motions

1050-1130 Tsuyoshi Ando (Hokkaido University)
Positivity, complete positivity and super-positivity

1130-1200 Hiroaki Yoshida (Ochanomizu University)
A remark on the conjugate variable and the free relative entropy of probability measures

LUNCH

1330-1400 Anna Kula (Jagiellonian University, Krakow, Poland)
Non-commutative Brownian motions on positive cones: Part I

1400-1430 Janusz Wysoczański (University of Wrocław, Poland)
Non-commutative Brownian motions on positive cones: Part II

1430-1500 Marie Choda (Osaka Kyoiku University)
Entropy for inner conjugacy class of subalgebras

1520-1550 Hun Hee Lee (Chungbuk National University, Korea)
An application of free probability to operator spaces

1550-1620 Nobuhiro Asai (Aichi University of Education)
The construction of probability measures related with the Jacobi-Szegö parameters

1620-1700 Masanao Ozawa (Nagoya University)
Classification of quantum measurements

September 12 (Sat) GSIS, Large Lecture Hall (2F)

0930-1010 Dénes Petz (Budapest University of Technology and Economics and Alfred Renyi Institute of Mathematics, Hungary)
Complementary subalgebras

1020-1100 Un Cig Ji (Chungbuk National University, Korea)
Quantum white noise derivatives and their applications to implementation problems

1100-1130 Masaya Tomie (Tsukuba University)
Möbius numbers of some modified generalized noncrossing partitions

LUNCH

1300-1340 Łukasz Wojakowski and Anna Krystek (University of Wrocław, Poland)
Infinitely divisible classes of measures in conditionally free probability

1340-1410 Hiromichi Ohno (Shinshu University)
Extremal marginal tracial states

1420-1450 Takuho Miyamoto (Tohoku University)
Free entropy dimension of projections and factoriality of generated von Neumann algebras

1450-1530 Wojciech Młotkowski (University of Wrocław, Poland)
Poisson measures in conditionally free independence

ABSTRACTS

Positivity, complete positivity and super-positivity

Tsuyoshi Ando ando@es.hokudai.ac.jp

For a linear map Φ from a unital C^*-subalgebra $\mathfrak{A}$ of $B(\mathcal{H})$ to $B(\mathcal{K})$, the notions of positivity and completely positivity are well known. Φ will be said to be *super-positive* if there is a positive linear map Ψ from $\mathfrak{A}$ to a *commutative* C^*-subalgebra of $B(\mathcal{G})$ for a Hilbert space $\mathcal{G}$ and $L \in B(\mathcal{K}, \mathcal{G})$ such that

$$\Phi(A) = L^* \cdot \Psi(A) \cdot L \quad (A \in \mathfrak{A}).$$

Obviously, super-positivity $\Longrightarrow$ complete positivity $\Longrightarrow$ positivity.

1. When $\mathfrak{A} = B(\mathbb{C}^n) = M_n(\mathbb{C})$ and $B(\mathcal{K}) = M_n(\mathbb{C})$, under natural identifications of the mapping space, to each positivity corresppponds each natural positive cone:

$$M_n(\mathbb{C}) \otimes M_m(\mathbb{C}) \simeq B\Big(\overset{n}{\underset{1}{\oplus}} \mathbb{C}^m\Big) \simeq B\Big(M_n(\mathbb{C}), M_m(\mathbb{C})\Big).$$

2. Under some verifiable conditions on Φ, complete positivity (resp. positivity) implies super-positivity (resp. complete positivity).
3. Mutual posititions of those three cones will be intuitively visualized.

The construction of probability measures related with the Jacobi-Szegö parameters

Nobuhiro Asai nasai@auecc.aichi-edu.ac.jp

It will be reported that probability measures on $\mathbb{C}$ associated with the Jacobi-Szegö parameters of the orthogonal polynomials can be constructed by making use of the classical Mellin convolution. Nontrivial probability measures expressed by the modified Bessel functions will be presented as examples.

Why normal law is infinitely divisible in the free probability? Connections with generalized Brownian motions

Marek Bożejko bozejko@math.uni.wroc.pl

We present the ideas of the proof of our last result with S. Belinschi, F. Lehner and R. Speicher that Normal law N(0,1) is infinitely divisible in the free additive free convolutions. The main steps are following:

1. Generalized Brownian motions and Wick formula.
2. Combinatorics of connected 2-partitions.
3. Analytic properties of R-transform of normal law.
4. Some remarks on others interesting free infinitely divisible measures.

Entropy for inner conjugacy class of subalgebras

Marie Choda marie@cc.osaka-kyoiku.ac.jp

Given a finite von Neumann algebra, we discuss about inner conjugacy classes of subalgebras from a view point of entopy.

Quantum white noise derivatives and their applications to implementation problems

Un Cig Ji uncigji@chungbuk.ac.kr

We study some linear differential equations associated with the quantum white noise derivatives and, as an application, implementation problems related to the Bogoliubov transformation and quentum extention of Girsanov theorem. (Jointly with Nobuaki Obata)

Infinitely divisible classes of measures in conditionally free probability

Anna Krystek Anna.Krystek@math.uni.wroc.pl
Łukasz Wojakowski Lukasz.Wojakowski@math.uni.wroc.pl

The conditionally free probability is dealing with noncommutative probability spaces with two states, and with the related convolution of pairs of measures. It includes as special cases the free and the boolean probabilities. We shall discuss the notion of conditionally free infinite divisibility and the analogues of classical infinitely divisible classes of measures.

Non-commutative Brownian motions on positive cones: Part I and II

Anna Kula Anna.Kula@im.uj.edu.pl
Janusz Wysoczański Janusz.Wysoczanski@math.uni.wroc.pl

We extend the Muraki's construction of the monotonic Brownian motions to the case of bm-independence. Also the bm-version of the Donsker Theorem is proved. This requires a reformulation of the bm-Central Limit Theorems so that it is related to intervals in the given positive cone. This is done for particular symmetric cones, including the Lorentz cone, the positive definite 2×2 real symmetric matrices and $(\mathbb{R}_+)^d$.

An application of free probability to operator spaces

Hun Hee Lee hhlee@chungbuk.ac.kr

In Banach space theory probabilistic tools are often used to characterize the geometry of Banach spaces. For example, the contraction principle of Rademacher varibles and comparison of Rademacher averages and gaussian averages are one of those tools. As a non-commutative analog of the above theory it is natural to expect that free probability would do the same role for the anaysis of operator spaces. In this talk we will explain a contraction principle of free Haar unitaries and a comparison of averages with respect to free Haar unitaries and free gaussians. As an application we will give a characterization of Pisier's OH (operator Hilbert) space.

Free entropy dimension of projections and factoriality of generated von Neumann algebras

Takuho Miyamoto miya@ims.is.tohoku.ac.jp

We examine the free entropy and free entropy dimension for projections, and obtain a sufficient condition for the factoriality of the von Neumann algebra generated by projections in terms of their free entropy dimension. This corresponds to Voiculescu's result for self-adjoint elements.

Poisson measures in conditionally free independence

Wojciech Młotkowski Wojciech.Mlotkowski@math.uni.wroc.pl

We are going to describe the class of Poisson-type limit measures with respect to conditional freeness.

Extremal marginal tracial states

Hiromichi Ohno h_ohno@shinshu-u.ac.jp

A state in a coupled system is called a marginal tracial state if the restrictions onto each systems are the unique tracial states. In this talk, we consider the rank of extremal marginal tracial states.

Classification of quantum measurements

Masanao Ozawa ozawa@math.cm.is.nagoya-u.ac.jp

We develop a multiplicity theory of completely-positive-map-valued measures using the multiplicity theory of type I von Neumann algebras and apply it to a classification of quantum measurements.

Complementary subalgebras

Dénes Petz petz@math.bme.hu

Maximal Abelian and factor subalgebras of finite dimensional C*-algebras are considered. The relation with mutually unbiased bases, generalizations of the Pauli quantum channels and the efficiency in quantum state estimation are the main subjects. Open problems and conjectures are also included.

Möbius numbers of some modified generalized noncrossing partitions

Masaya Tomie tomie@math.tsukuba.ac.jp

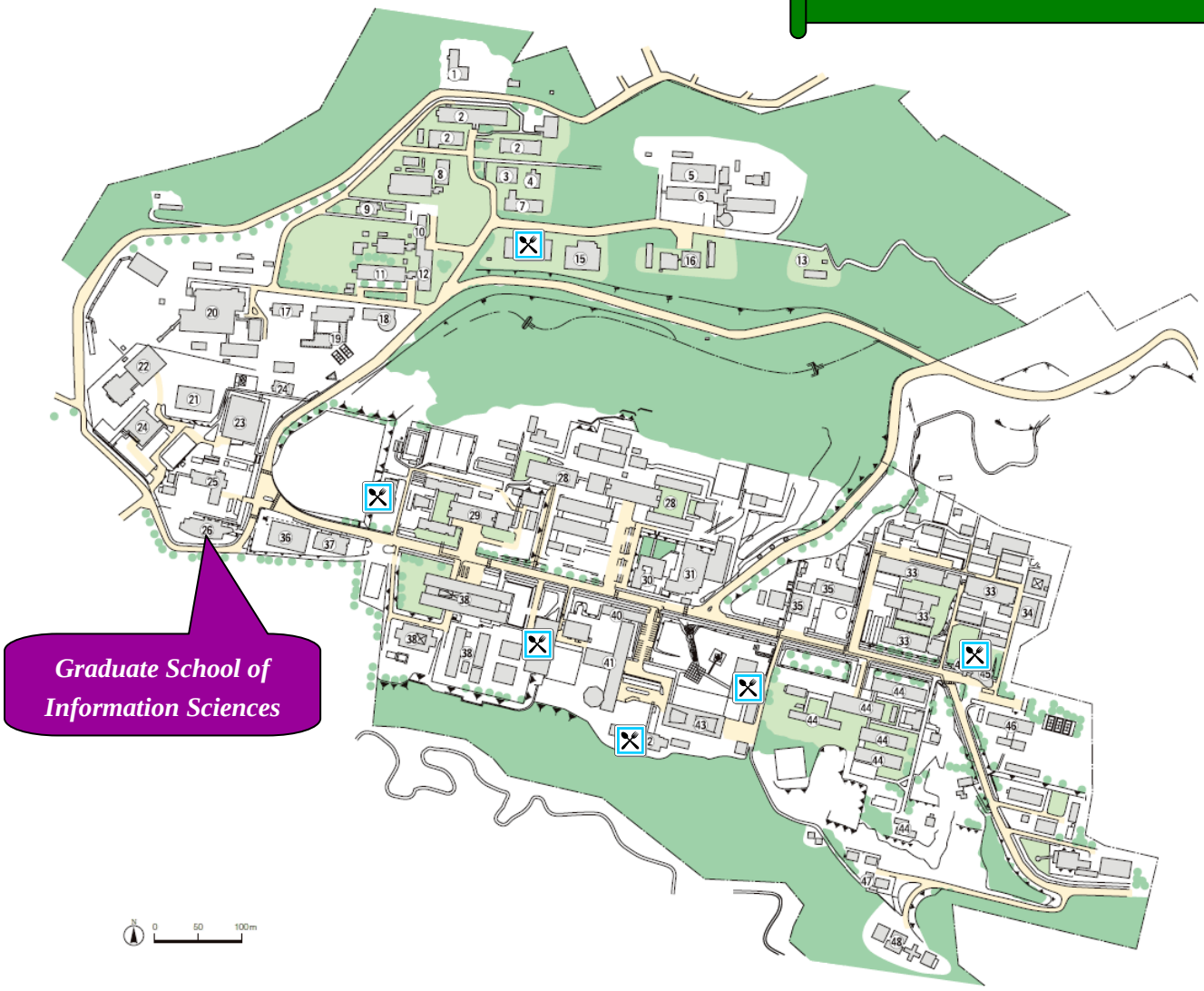
In this talk we consider the Möbius number of truncated generalized noncrossing partitions. Then we show that they are described using the positive Fuss–Catalan numbers which is a generalization of the Catalan numbers.

A remark on the conjugate variable and the free relative entropy of probability measures

Hiroaki Yoshida yoshida@edu.is.ocha.ac.jp

Recently a new estimation theoretic representation for the classical relative entropy of probability measures has been given in terms of MMSE (minimum mean square error) by Verdué and he has also applied this representation to the free case. We will see that, in free case, this representation can be formulated by Voiculescu's conjugate variables.

Aobayama Campus Map



Organizing Committee

Nobuaki Obata (Chair, Tohoku University) obata@math.is.tohoku.ac.jp
Fumio Hiai (Co-Chair, Tohoku University) hiai@math.is.tohoku.ac.jp
Marek Bożejko (University of Wrocław) bozejko@math.uni.wroc.pl
Un Cig Ji (Chungbuk National University) uncigji@chungbuk.ac.kr

Sponsored by

JSPS-PAN Joint Research Project (2008-2009)
"Noncommutative Harmonic Analysis on Discrete Structures with Applications to Probability"
JSPS-HAS Joint Research Project (2008-2009)
"New Developments in Quantum Information Theory and Quantum Probability Theory"