2016 年度後期

確率モデル論 (情報科学研究科)
応用解析学 (工学研究科)
確率モデル論 (国際高等研究教育院)

● 授業科目の目的・概要及び達成目標等

ランダム現象の数理解析のための基礎知識として、確率モデルの構成とその解析手法を学ぶ。確率論の基礎概念（確率変数・確率分布など）から始めて、ランダム現象の時間発展を記述する確率過程として、ランダムウォーク・マルコフ連鎖・マルコフ過程の典型例をとりあげて、その性質と幅広い応用を概観する。講義では学部初年級の確率統計の知識を前提とする。

As an introduction to mathematical analysis of random phenomena we learn probability models, their construction and analysis. We start with fundamental concepts in probability theory (random variables, probability distributions, and so on). For the time evolution of random phenomena we study basic properties of random walks, Markov chains, Markov processes, and take a bird's-eye view of their wide applications. Background knowledge on elementary probability is required.

● **Topics**

1. Random variables and probability distributions
2. Bernoulli trials
3. Random walks
4. Markov chains
5. Poisson processes
6. Queues
7. Galton-Watson branching processes
8. Birth-and-death processes .... etc

● 成績

(1) 配布プリントの問題 (**Problem** として通し番号がつく) から数題を選択してレポートを作成し, 1 月後半に提出してもらう予定. 詳細は後日発表する (コピーレポートは零点).

(2) 例外的な取り扱いは一切しない.

● Résumé

1. レジュメ (英語) を担当者のウェッブサイト (www.math.is.tohoku.ac.jp/˜obata) からダウンロードして, 各自準備する.
   Download the materials from Obata's website upon participating in the lectures.
2. ウェッブサイトには, 過去の講義録・その他の資料も掲載されている.
3. レジュメは毎回の講義内容の項目を示すものでテキストではないから, レジュメだけ読んでも勉強にはならない. 書物をひも解いて勉強すること.
4. レジュメは講義の進度に従って準備するので, まとめて事前に配布することはしない.

● Basic References

1. 拙著：確率モデル要論 (牧野書店), 2012.
   過年度の講義をまとめたもので, 本講義の内容もおおむねこの本にしたがう.
2. D. L. Minh: Applied Probability Models, Duxbury, 2001.
   英語で読むのなら, この本が便利である.

● Further Reading

1. S. M. Ross: Introduction to Probability Models, 11th Ed. Academic Press, 2014.

   初版は 1972 年のロングセラー. だが, 800 ページに迫る大部. 取り扱っている内容は [Minh] とオーバーラップする部分が多いが、より初等的なレベルから解説していて読みやすい.

2. J. R. Norris: Markov Chains, Cambridge UP, 1998.

   いろいろな具体例を扱い, 大変読みやすい定評のある教科書.

3. W. Feller: An Introduction to Probability Theory and Its Applications, Vol. 1, Wiley, 1957.

   名著の誉れ高い. この本は講義内容をカバーし, さらに詳しいことがたくさん書かれている (Vol. 2 もある!). 邦訳もある.

   W. フェラー (河田龍夫他訳) : 確率論とその応用 (紀伊国屋). こちらは 4 分冊.

4. B. V. Gnedenko: The Theory of Probability and the Elements of Statistics, AMS Chelsea Publishing Co., 6th ed. 1989.

5. R. Durrett: Probability: Theory and Examples, Duxbury Press, 1996.

   この 2 冊も講義内容をカバーし, さらに詳しいことがたくさん書かれている. この程度の知識があれば, 確率モデルを本格的に研究に生かせるだろう.

6. 志賀徳造：ルベーグ積分から確率論 (共立), 2000.

   前半はルベーグ積分を展開しているが, 後半でランダムウォークを取り扱って確率モデルへの入門をはたす.

7. R. B. シナジ (今野紀雄・林俊一訳) : マルコフ連鎖から格子確率モデルへ, シュプリンガー東京, 1999.

   マルコフ連鎖の基礎理論, ゴルトン・ワトソン過程, 出生死亡過程を含み, 手頃である.

8. 国沢清典 : 確率論とその応用 (岩波全書), 1982.

   少し古いが, 本講義はこのレベルをめざす.

9. 舟木直久：確率論, 朝倉書店, 2004.

10. 西尾真喜子：確率論, 実教出版, 1978.

    この 2 冊はさらに高度なところまで数学理論として展開している.

11. P. ブレモー (釜江哲朗監修, 向井久訳)：モデルで学ぶ確率入門 (新装版), シュプリンガー東京, 2004.

    実用の場面を想定したさまざまな確率モデルが取り上げられている. 例題を通して数学的な枠組を学ぶ形式で書かれている. 個々の事例は興味深いが, 理論を知った上で見ないと難しいかもしれない.

12. F. Spitzer: Principles of Random Walk, Springer, 2nd Ed., 1976.

    ランダムウォークに関するプロ向きの本.

13. K. L. Chung: Markov Chains, Springer, 1960.

    マルコフ連鎖に関するプロ向きの本.

14. イアン・ハッキング (石原英樹・重田園江訳)：偶然を飼いならす, 木鐸社, 1999.

    「この博物誌的な書物を好奇心に満ちたすべての読者に捧げる」とある. 確率統計が 20 世紀の科学に中でいかに成功してきたかを科学史的な視点で論ずる. かなり興味深い.

15. イアン・ハッキング (広田すみれ・森元良太訳)：確率の出現, 慶應義塾大学出版会, 2013.

16. キース・デブリン (原 啓介訳) : 世界を変えた手紙 — パスカル、フェルマーと〈確率〉の誕生, 岩波書店, 2010.

    確率論の始まりをさまざまなエピソードとともに語る. 個人的にはカルダーノに大変興味がある.

Overview

0.1 Stochastic Processes

We will study the probability models for time evolution of random phenomena. Measuring a certain quantity of the random phenomenon at each time step $n = 0, 1, 2, \dots$, we obtain a sequence of real values:

$$x_0, x_1, x_2, \dots, x_n, \dots.$$

Because of randomness, we consider x_n as a realized value of a random variable X_n. Here a random variable is a variable taking several different values with certain probabilities. Thus, the time evolution of a random phenomenon is modeled by a sequence of random variables

$$\{X_n\,;\, n = 0, 1, 2, \dots\} = \{X_0, X_1, X_2, \dots, X_n, \dots\},$$

which is called a *discrete-time stochastic process*. If the measurement is performed along with continuous time, we need a *continuous-time stochastic process*:

$$\{X_t\,;\, t \geq 0\}$$

It is our purpose to construct stochastic processes modeling typical random phenomena and to demonstrate their properties within the framework of modern probability theory.

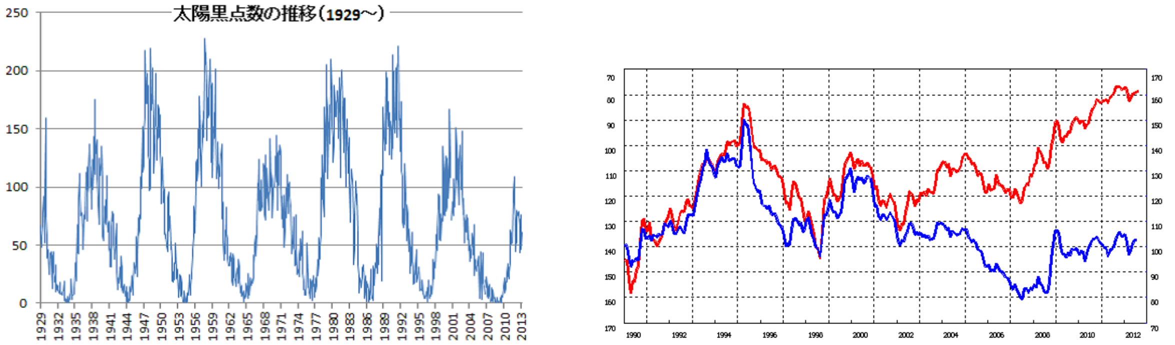


Figure 1: Solar spots; Nominal exchange rate (red) and real effective exchange rate (blue)

We hope that you will obtain basic concepts and methods through the following three subjects:

0.2 One-Dimensional Random Walk and Gambler's Ruin Problem

Let us consider coin tossing. We get $+1$ if the heads appears, while we get -1 (i.e., lose $+1$) if the tails appears. Let Z_n be the value of the n-th coin toss.

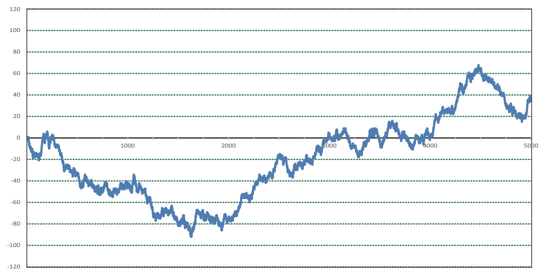
To be precise, we must say that $\{Z_n\}$ is a sequence of independent, identically distributed (iid) random variables with the common distribution

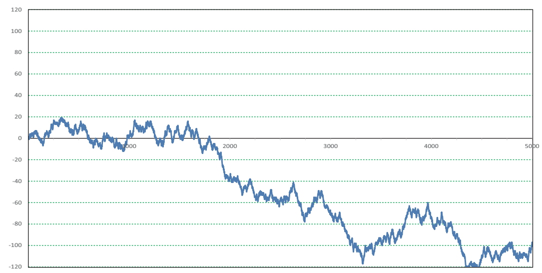
$$P(Z_n = +1) = P(Z_n = -1) = \frac{1}{2}.$$

In short, $\{Z_n\}$ is called the *Bernoulli trials* with success probability $1/2$. Define

$$X_0 = 0, \quad X_n = \sum_{k=1}^{n} Z_k \quad n = 1, 2, \dots.$$

Then X_n means the net income at the time n, or the coordinate of a drunken walker after n steps. The discrete time stochastic process $\{X_n\}$ is called *one-dimensional random walk*.





(i) law of large numbers

(ii) diffusion speed (central limit theorem)

(iii) recurrence

(iv) long leads (law of happy time)

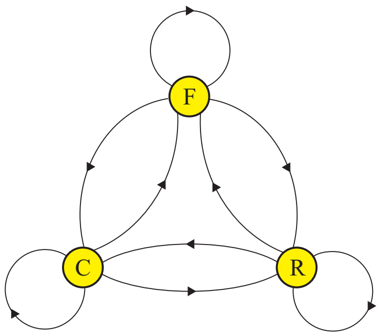
(v) gambler's ruin (random walk with barriers)

0.3 Markov Chains

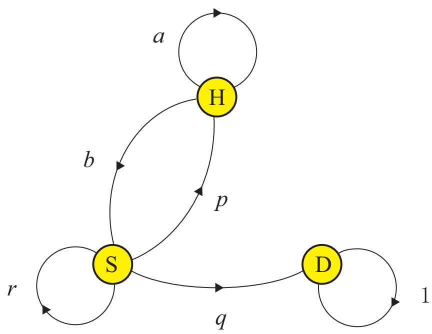
Consider the time evolution of a random phenomenon, where several different *states* are observed at each time step $n = 0, 1, 2, \dots$. For example, for the ever-changing weather, after simplification we observe three states: fine, cloudy, rainy. Collected data look like a sequence of F, C, R:

$$F\,F\,C\,R\,F\,C\,C\,F\,R\,\dots$$

from which we may find the conditional probability $P(X|Y)$ of having a weather X just after Y. Then we come to the transition diagram, where each arrow $Y \to X$ is asigned the conditional probability $P(X|Y)$.



The above diagram describes a general Markov chain over the three states because the transitions occur between every possible pair of states. According to our purpose, we may consider variations. For example, we may consider the following diagram for analysis of life span.

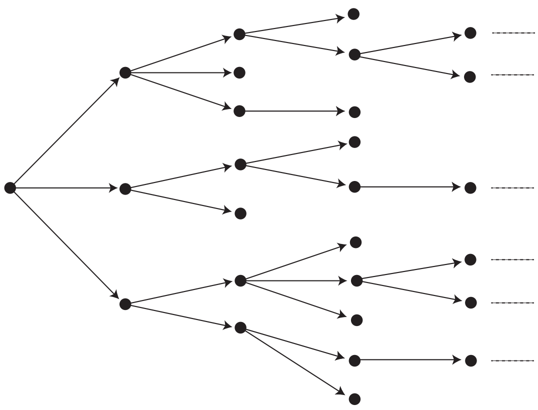


A Markov chain $\{X_n\}$ is a discrete-time stochastic process over a state space $S = \{i, j, \dots\}$ (always assumed to be finite or countably infinite), which is governed by the one-step transition probability:

$$p_{ij} = P(X_{n+1} = j | X_n = i)$$

where the right hand side is independent of n (time homogeneous). A random walk is an example of a Markov chain. The theory of Markov chains is one of the best successful theories in probability theory for its simple description and unexpectedly rich structure. We are interested in the following topics:

(i) stationary distribution
(ii) recurrence
(iii) average life span
(iv) survival of family names (Galton-Watson tree)
(v) birth-and-death chains



0.4 Poisson Process

Let us imagine that an event E occurs repeatedly at random as time goes on. For example, alert of receiving an e-mail, passengers making a queue at a bus stop, customers visiting a shop, occurrence of defect of a machine, radiation from an atom, etc.

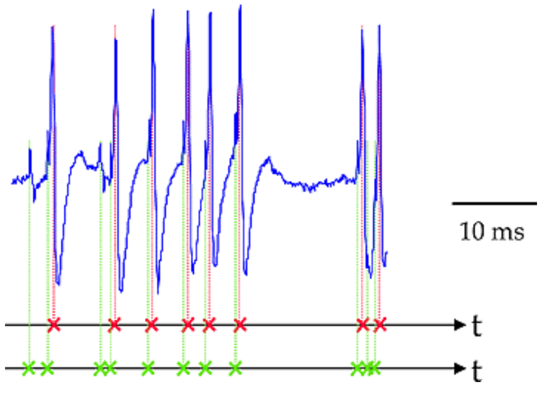
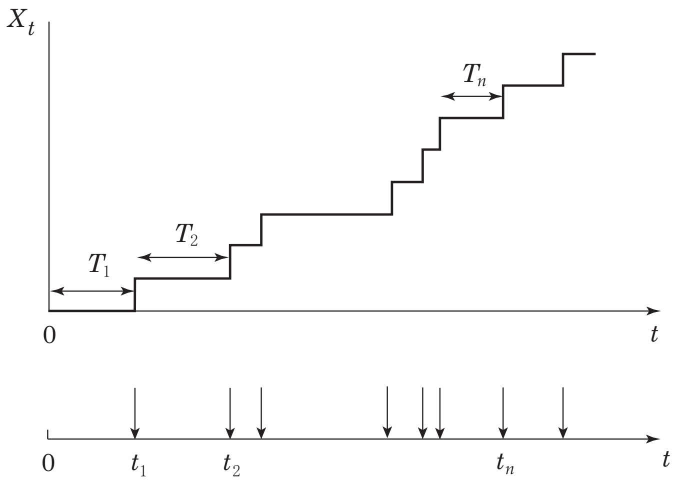


Figure 2: Nerve impulses

To obtain a stochastic process, we count the number of occurrence of the event E during the time interval $[0, t]$, which is denoted by X_t. Then we obtain a stochastic process $\{X_t ; t \geq 0\}$. The situation is illustrated as follows, where $t_1, t_2, \dots$ are the time when E occurs.



A fundamental case is described by a Poisson process, where the event happens independently each other. The first to check is the statistics between two consecutive occurrence of events (waiting time).

(i) Applications to queuing theory (waiting lines are modeled by a Poisson process).

(ii) A birth-and-death process as generalization.

Poisson process is one of the fundamental examples of (continuous-time) *Markov processes*. Another is the Brownian motion (Wiener process).

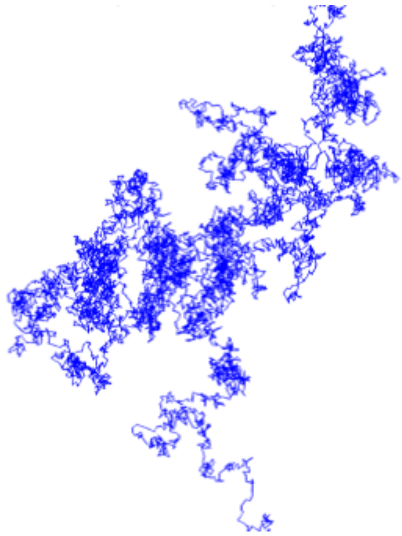


Figure 3: Two-dimensional Brownian Motion (simulation)