Masters Thesis

Discrete time population dynamics model for exploitative competition between native and alien predators

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1 Introduction

Today numerous species are experiencing habitat shifts potentially driven by climate change. Invasion of an alien predator species into a native prey-predator system could lead to substantial ecological consequences. In this research we present a discrete-time population dynamics model to analyze exploitative competition between native and alien predators sharing a common prey. The study investigates the conditions under which an alien predator can successfully invade a native predator-prey system, the persistence of both predators, and the consequences of their interactions.

2 Assumptions

- The predator is a specialist, dependent solely on a specific prey.
- The native prey-predator environment is invaded by an alien predator.
- The predation is stage-specific.
- Adults of prey and predator die out after their respective reproductive season.
- Prey population features density-dependent intraspecific competition.

3 Model J

We consider a prey-predator interaction where the native predator is P, which targets the juvenile stage of the prey population and Q is the invading predator which targets the adult prey. The native prey-predator system of Model J is as follows

$$\begin{cases} H_{n+1} &= e^{-a_1 P_n} \dfrac{r_0}{1 + H_n/\beta} H_n, \\ P_{n+1} &= \rho_1 \left(1 - e^{-a_1 P_n}\right) \dfrac{r_0}{1 + H_n/\beta} H_n. \end{cases}$$

where H_n and P_n are the adult prey and predator population densities respectively, at generation n. r_0 is the supremum number of offsprings produced by an adult prey, ρ_1 is the reproduction rate of the predator per prey consumed for P, a_1 is the attack efficiency of the juvenile and β is the coefficient of the strength of density effect for the reproduction in the predator population.

4 Model A

Contrary from Model J, in Model A the native predator is Q, which targets the adult prey and the invading predator is P, which targets the juvenile prey. The equations for the native prey-predator system of Model A are as follows

$$\begin{cases} H_{n+1} &= \dfrac{r_0}{1 + e^{-a_2 Q_n} H_n/\beta} e^{-a_2 Q_n} H_n, \\ Q_{n+1} &= \rho_2 \left(1 - e^{-a_2 Q_n}\right) H_n. \end{cases}$$

where Q_n is the adult predator population density for Q at generation n. ρ_2 is the reproduction rate of the predator per prey consumed for Q, a_2 is the attack efficiency of the predator.

5 Dynamics with invading alien predator

We extend the dynamics of the native prey-predator model to incorporate an invading alien predator, resulting in a three-species system: the prey H, the predator P, and Q.

$$\begin{cases} H_{n+1} = e^{-a_1 P_n} \dfrac{r_0}{1 + e^{-a_2 Q_n} H_n/\beta} e^{-a_2 Q_n} H_n, \\ P_{n+1} = \rho_1 \left(1 - e^{-a_1 P_n}\right) \dfrac{r_0}{1 + e^{-a_2 Q_n} H_n/\beta} e^{-a_2 Q_n} H_n, \\ Q_{n+1} = \rho_2 \left(1 - e^{-a_2 Q_n}\right) H_n. \end{cases}$$

To simplify analysis, we non-dimensionalize the system by introducing scaled variables for prey and predator densities, as well as new parameters. Let

$h_n = \frac{H_n}{\beta}$, $p_n = a_1 P_n$, $q_n = a_2 Q_n$, $\alpha_1 = a_1 \rho_1 \beta$, $\alpha_2 = a_2 \rho_2 \beta$.

$$\begin{cases} h_{n+1} = e^{-p_n} \dfrac{r_0}{1 + e^{-q_n} h_n} e^{-q_n} h_n, \\ p_{n+1} = \alpha_1 \left(1 - e^{-p_n}\right) \dfrac{r_0}{1 + e^{-q_n} h_n} e^{-q_n} h_n, \\ q_{n+1} = \alpha_2 \left(1 - e^{-q_n}\right) h_n. \end{cases}$$

6 Predator persistence in the native prey-predator system

When $r_0 < 1$ the prey population goes to extinction. Hence. we will consider the case for $r_0 > 1$. For the native prey-predator system of Model J, we have

$$p_{n+1} = \alpha_1 \left(1 - e^{-p_n}\right) \frac{r_0}{1+h_n} h_n < \alpha_1 \left(1 - e^{-p_n}\right)(r_0 - 1),$$

for any $n > 0$ and $h_n > 0$. Then $p_n \to 0$ as $n \to \infty$ if $R_P^0 \leq 1$. Similarly, for the native prey-predator system of Model A, we have

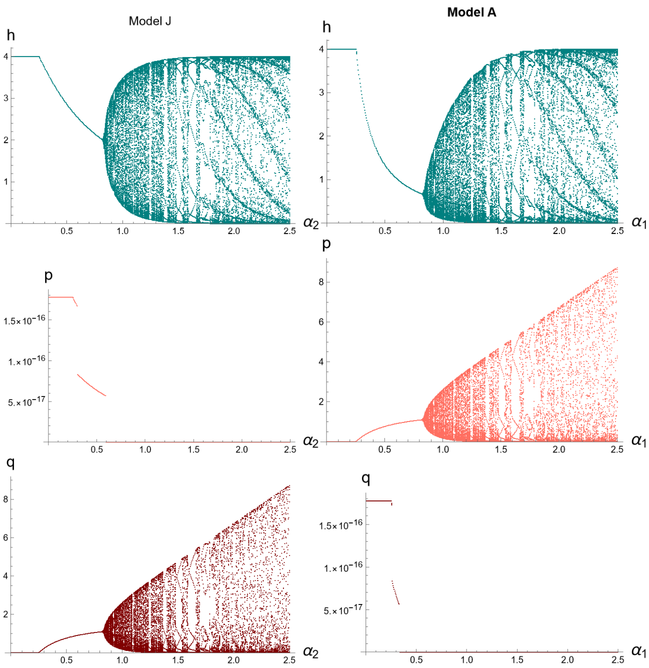
$$q_{n+1} = \alpha_2 \left(1 - e^{-q_n}\right) h_n < \alpha_2 \left(1 - e^{-q_n}\right)(r_0 - 1),$$

where $q_n \to 0$ as $n \to \infty$ if $R_Q^0 \leq 1$. As a consequence, these results indicate the global stability of the predator extinction equilibrium for the native prey-predator system of Model J and Model A with $\mathscr{R}_0^P \leq 1$ and $\mathscr{R}_0^Q \leq 1$ respectively where

$$\mathscr{R}_0^{\mathrm{P}} = \alpha_1(r_0 - 1),$$
$$\mathscr{R}_0^{\mathrm{Q}} = \alpha_2(r_0 - 1).$$

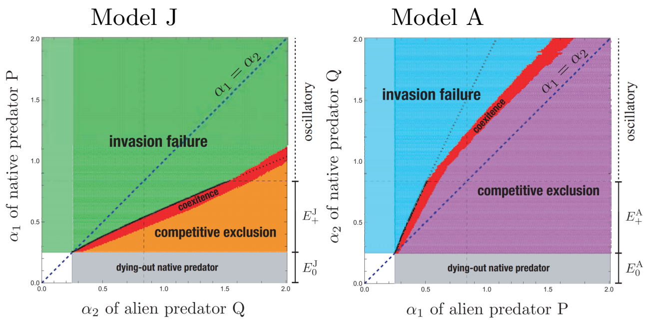
Predator persistence is the case when the native predator persists in the environment without going extinct which is possible when $\mathscr{R}_0^P > 1$ for native prey-predator system of Model J and $\mathscr{R}_0^Q > 1$ for native prey predator system of Model A.

7 Invasion success of the alien predator



This numerical example of the bifurcation shows different regions where invasion is successful. In the case of Model J, initially it can be observed that the alien predator Q is unable to invade in the native prey-predator system where the prey H and predator P exist in equilibrium. After that, the invasion is successful and both the predators Q and P coexist. However, this region is small. After that the native predator P goes extinct, leading to competitive exclusion where the alien predator Q exists in a stable state with the H. Similar observations can be made for Model A where P is the invading predator and Q is the native predator.

8 Invadability



This result indicates that the invasion of predator Q in Model J, that is the native prey-predator system with predator P, is harder to be successful than that of predator P in Model A, that is the native prey-predator system with predator Q. Especially when $\alpha_1 = \alpha_2$ for predators P and Q, we can see that the invasion of predator P in the native prey-predator system of Model A is successful, while that of predator Q in the native prey-predator system of Model J is unsuccessful. In Model J, α_2 needs to be sufficiently larger than α_1 to invade successfully. However, in the case of Model A, α_1 can be lesser than α_2 and still invade successfully. Coexistence is possible but difficult.

9 Concluding remarks

These findings underscore the critical role of life stage-specific interactions in determining the outcomes of species invasions. Future research may extend these models to account for additional ecological factors such as environmental variability, multi-predator interactions, and adaptive behaviors in prey and predators.