数理モデリングの理路が大事

感染症伝染・情報伝播・行動依存症蔓延の数理モデリング

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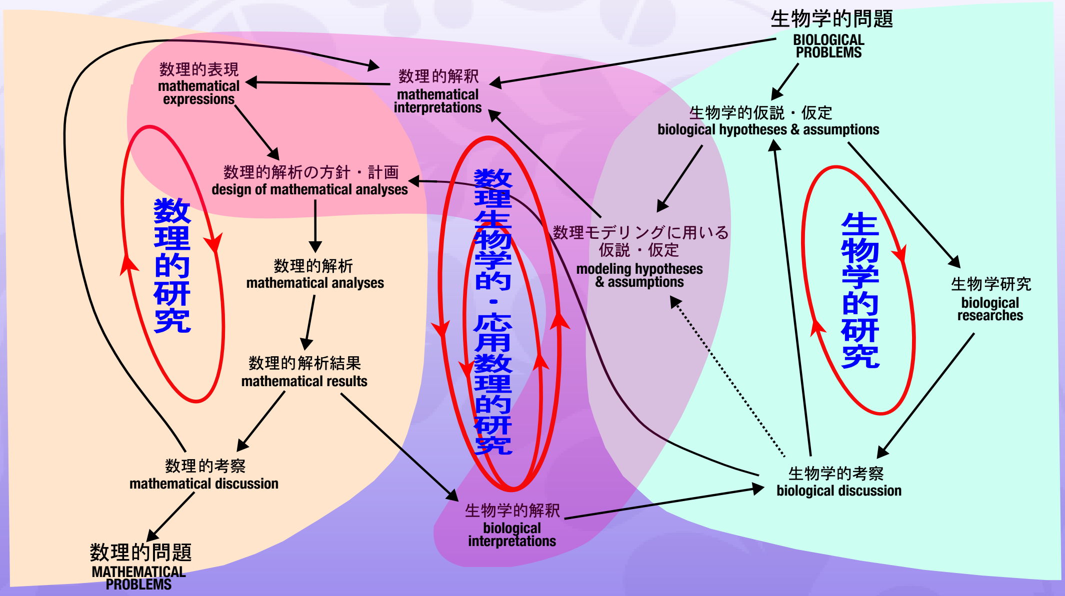
Prologue



生物学的問題
BIOLOGICAL PROBLEMS
数理的表現
mathematical expressions
数理的解釈
mathematical interpretations
生物学的仮説・仮定
biological hypotheses & assumptions
数理的解析の方針・計画
design of mathematical analyses
数理的研究
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modeling hypotheses & assumptions
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biological discussion
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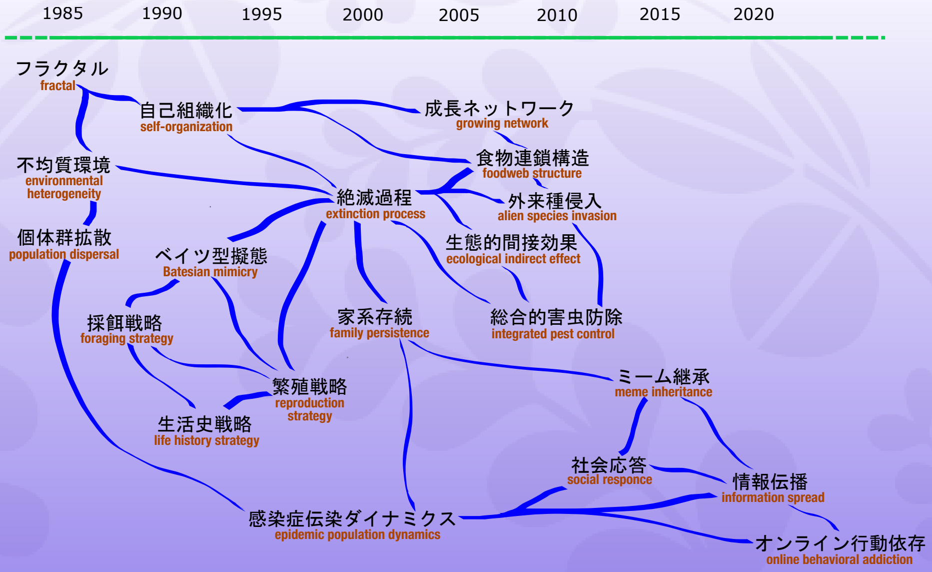
数理的問題
MATHEMATICAL PROBLEMS

Prologue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オンライン行動依存
online behavioral addiction

Population dynamics model

Prologue

Population Dynamics

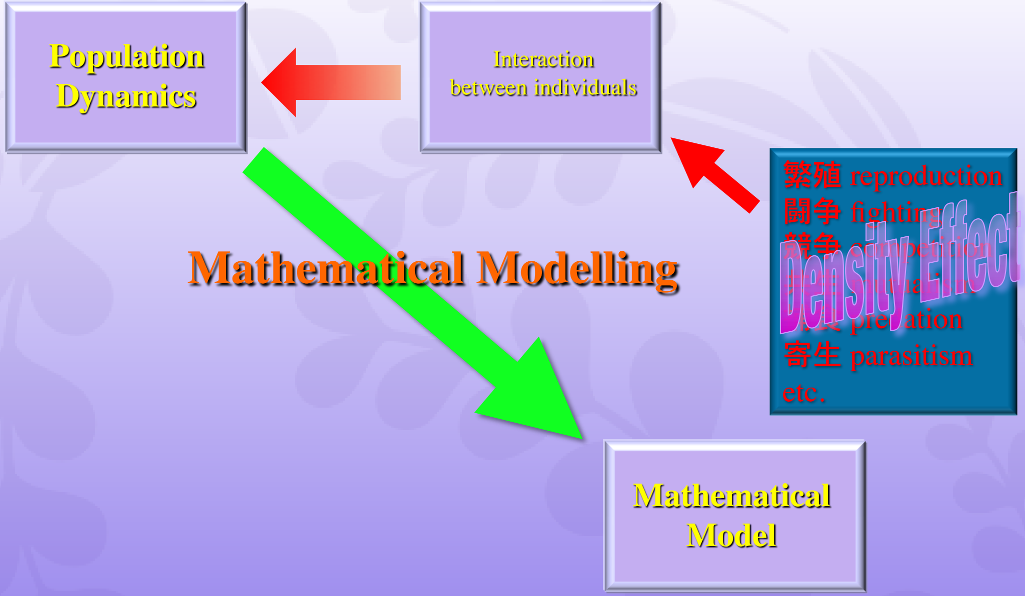
is the nature of the spatio-temporal variation in biological population size (density etc.).

繁殖 reproduction
闘争 fighting
競争 competition
共生 mutualism
捕食 predation
寄生 parasitism
etc.

Mathematical Modelling

Mathematical Model

Prologue


Population Dynamics
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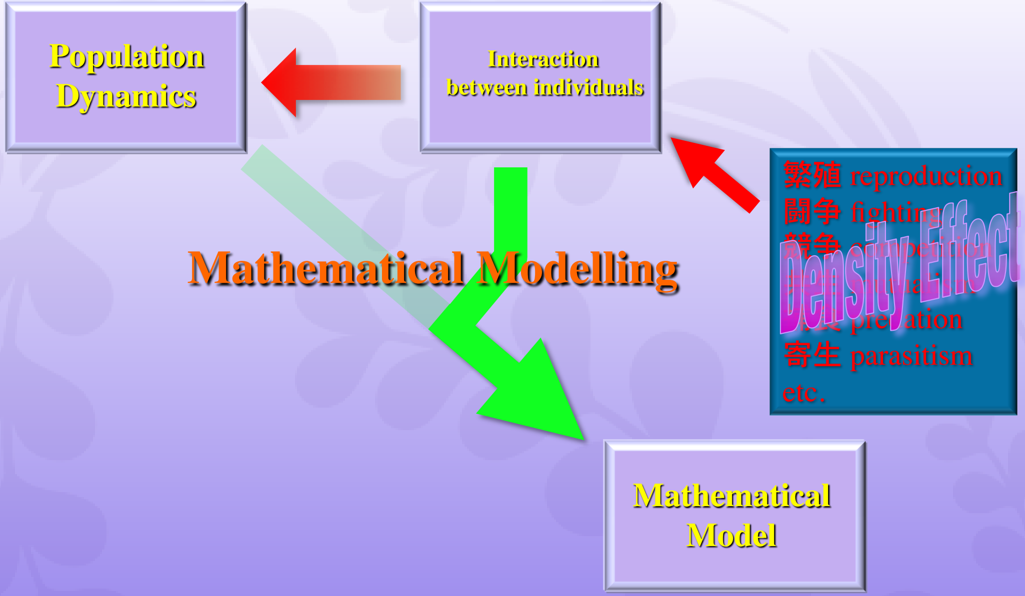
etc.
Density Effect
Mathematical Model

Prologue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etc.
Density Effect
Mathematical Model

What mathematical model is reasonable from the biological viewpoint?

What mathematical structure is appropriate for the reasonable modeling?

Distinct four logistic equations

[L-1] $$\frac{dN(t)}{dt} = \{r_0 - \beta N(t)\}N(t)$$

[L-2] $$\frac{dN(t)}{dt} = r_0\left\{1 - \frac{N(t)}{K}\right\}N(t)$$

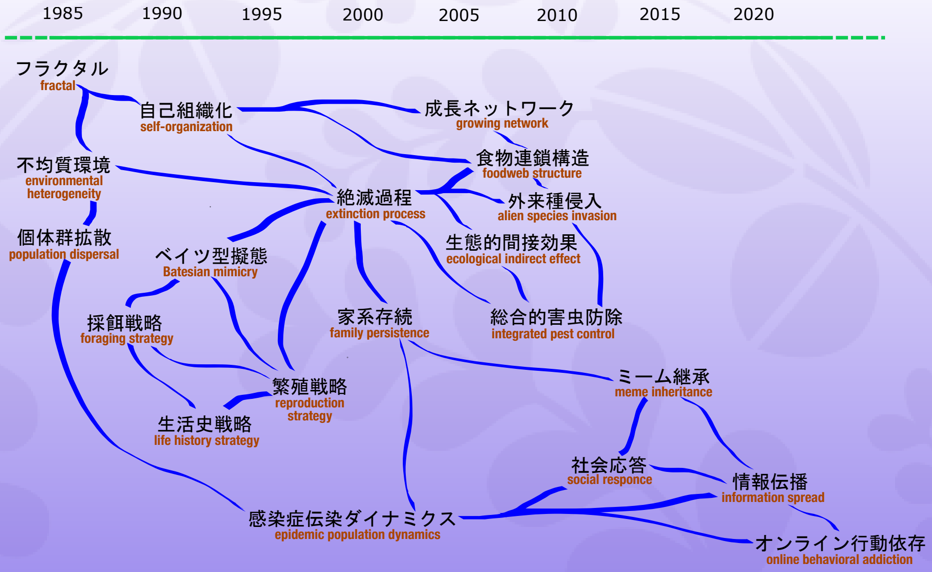
[L-3] $$\frac{dN(t)}{dt} = r_0 N(t) - b\{N(t)\}^2$$

[L-4] $$\frac{dN(t)}{dt} = \{r_0 - \beta N(t)\}N(t) - b\{N(t)\}^2$$

Reasonability of modeling depends on

i) purpose of theoretical research;
ii) available data/knowledge/hypothesis;
iii) design of mathematical analysis.

Prologue



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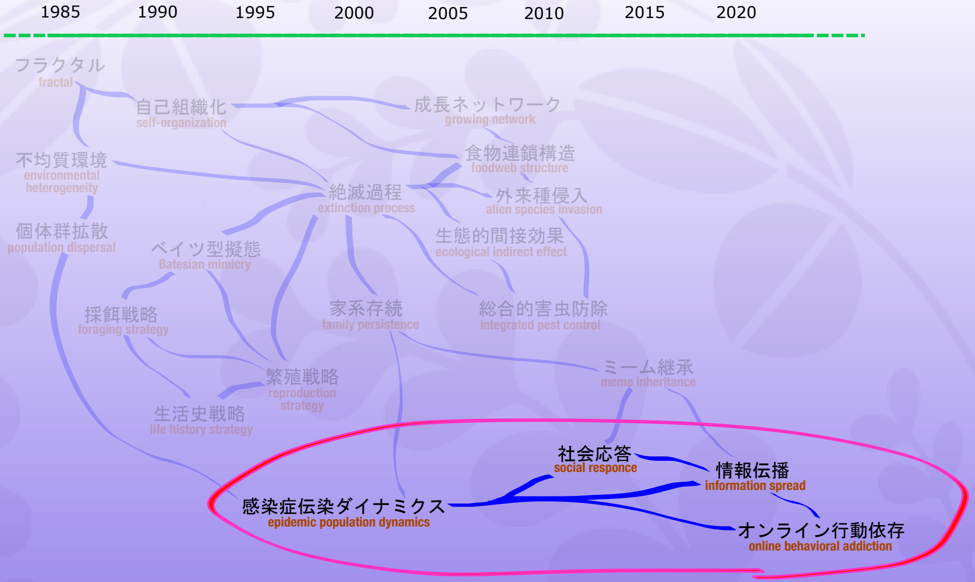
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Outline

Prologue

Population dynamics modeling for epidemics

Population dynamics modeling for information spread

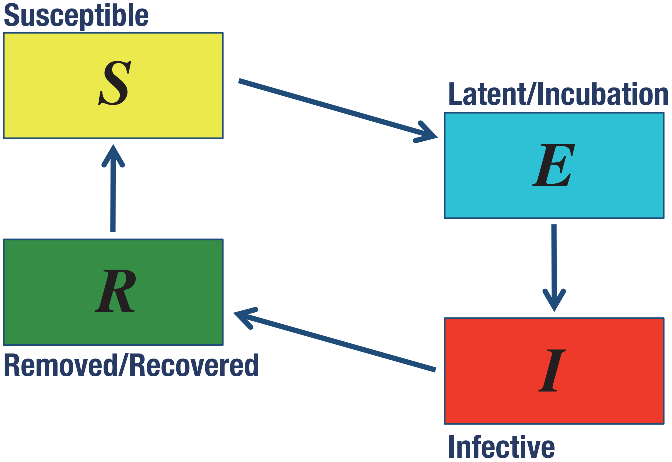
Population dynamics modeling for online behavioral addiction

Epilogue

Epidemics

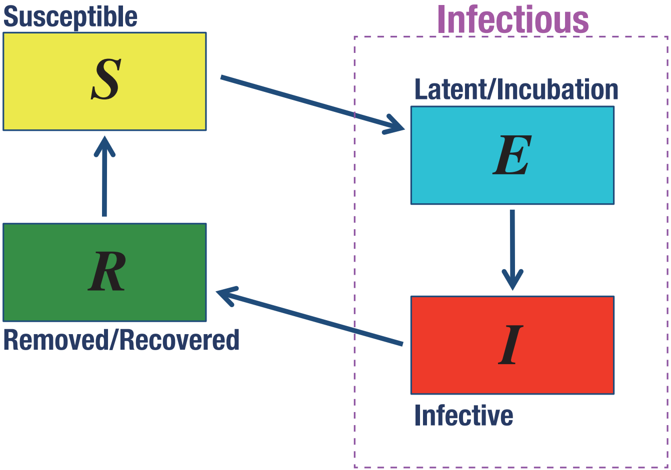
Epidemic dynamics of infectious disease

$S \to E \to I \to R \to S$



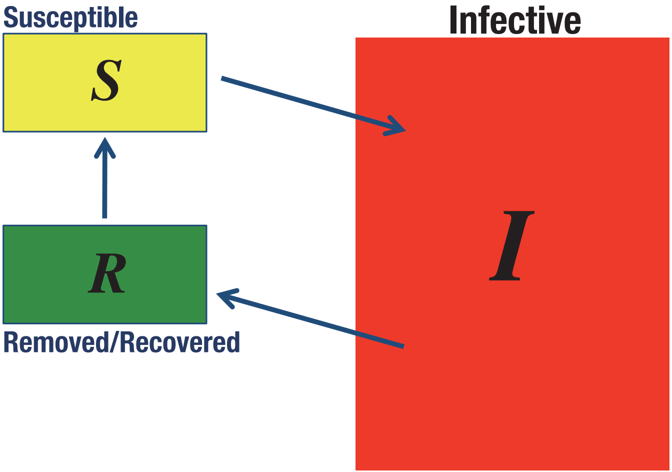
Epidemic dynamics of infectious disease

$S \longrightarrow E \longrightarrow I \longrightarrow R \longrightarrow S$



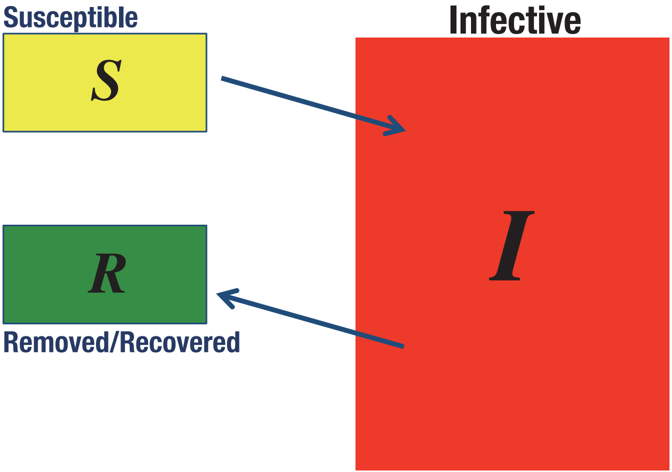
Epidemic dynamics of infectious disease

$S \to I \to R \to S$



Epidemic dynamics of infectious disease

$S \rightarrow I \rightarrow R$



Epidemic dynamics model

Generic SIR model

$$\frac{dS(t)}{dt} = B - \Lambda S(t) \qquad - \mu_{\mathrm{S}} S(t)$$

$$\frac{dI(t)}{dt} = \qquad \Lambda S(t) - qI(t) - \mu_{\mathrm{I}} I(t)$$

$$\frac{dR(t)}{dt} = \qquad qI(t) - \mu_{\mathrm{R}} R(t)$$

$\Lambda = \Lambda(S, I, R)$: Force of infection

Epidemic dynamics model

Generic SIR model without demographic change

$$\frac{dS(t)}{dt} = B - \Lambda S(t) - \mu_S S(t)$$

$$\frac{dI(t)}{dt} = \Lambda S(t) - qI(t) - \mu_I I(t)$$

$$\frac{dR(t)}{dt} = qI(t) - \mu_R R(t)$$

$$\frac{d}{dt}\{S(t) + I(t) + R(t)\} = 0; \quad S(t) + I(t) + R(t) = N$$

(time-independent constant total population size)

Kermack-McKendrick SIR model

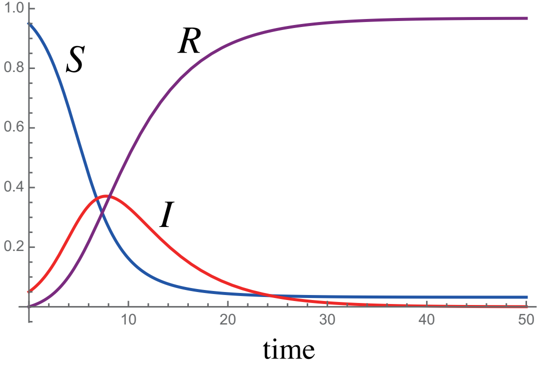
$$\Lambda \propto I$$

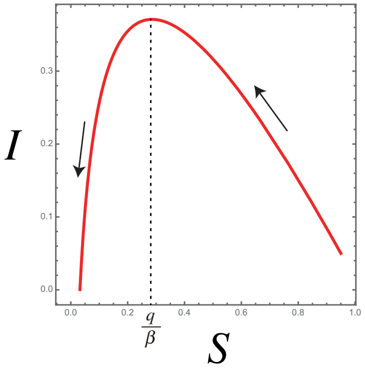
$$\frac{dS(t)}{dt} = -\beta I(t) S(t)$$

$$\frac{dI(t)}{dt} = \beta I(t) S(t) - qI(t)$$

$$\frac{dR(t)}{dt} = qI(t)$$

Kermack-McKendrick SIR model





Epidemic dynamics model

Essential factors for epidemic dynamics

- Route of disease transmission;
- Public health condition;
- Availability of medical services;
- Cultural/social customs in daily life;
- Social responses originated with cultural/political/economic background.

Epidemic dynamics model

Seno, H., *A Primer on Population Dynamics Modeling: Basic Ideas for Mathematical Formulation*, Springer Nature Singapore, Singapore, 2022.

Inaba, H., *Age-Structured Population Dynamics in Demography and Epidemiology*, Springer, Singapore, 2017.

齋藤保久・佐藤一憲・瀬野裕美, 数理生物学講義【展開編】 — 数理モデル解析の講究 —, 共立出版, 東京, 2017.

Martcheva, M., *An Introduction to Mathematical Epidemiology*, Texts in Applied Mathematics 61, Springer, New York, 2015.

Diekmann, O., Heesterbeek, H., Britton, T., *Mathematical Tools for Understanding Infectious Disease Dynamics*, Princeton University Press, Princeton, 2013.

日本数理生物学会（編），『数』の数理生物学（瀬野裕美 責任編集），シリーズ数理生物学要論, 第１巻, 共立出版, 東京, 2008.

稲葉寿, 感染症の数理モデル, 培風館, 東京, 2008.

Information spread

Information spread

Rumor, Fake news

Fashion, Trend, Vogue

Thought, Idea, Belief, Fear, Panic

Innovation, Culture, Habit, Language, Dialect

Parasite, Disease, Gene

etc.

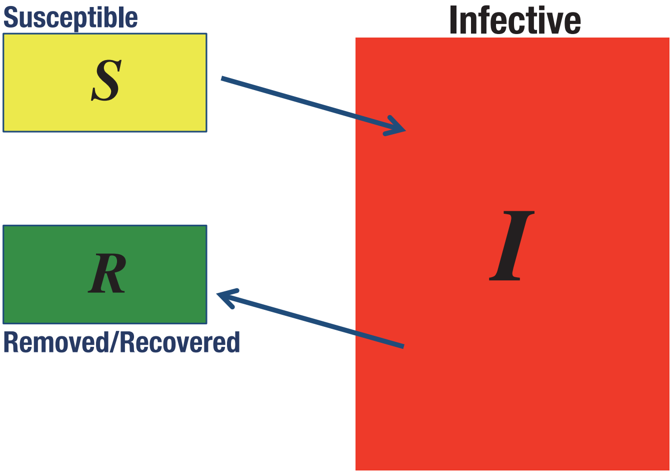
Mathematical modeling

- Stochastic process
- Network dynamics
- Cellular automaton
- Reaction-diffusion system
- Game-theoretic approach

etc.

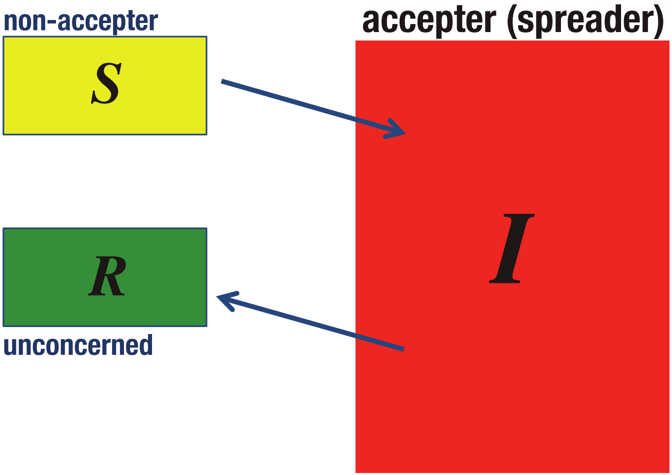
Information spread dynamics model

$S \to I \to R$



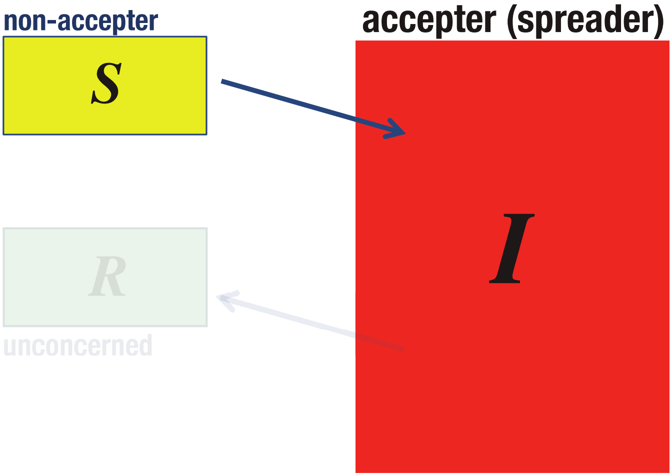
Information spread dynamics model

$S \to I \to R$



Information spread dynamics model

$S \rightarrow I \rightarrow R$



Granovetter's threshold model

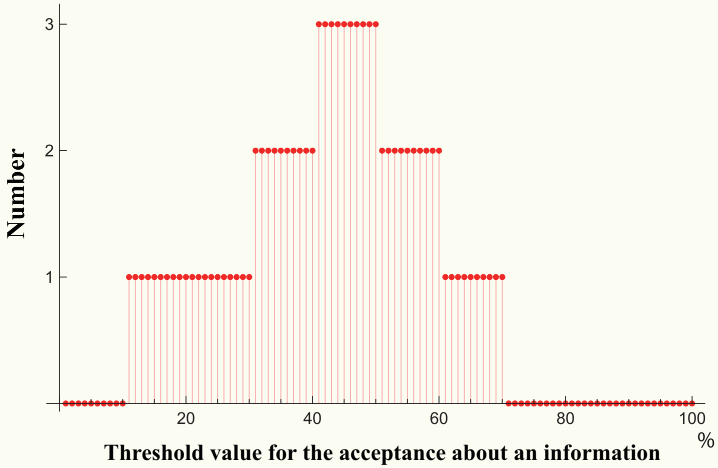
Granovetter, M. (1978) American Journal of Sociology, 83(6), 1420–1443.

Granovetter, M. (1983) Sociological Theory, 1, 201–233.

Granovetter, M., Soong, R. (1983) Journal of Mathematical Sociology, 9(3), 165–179.

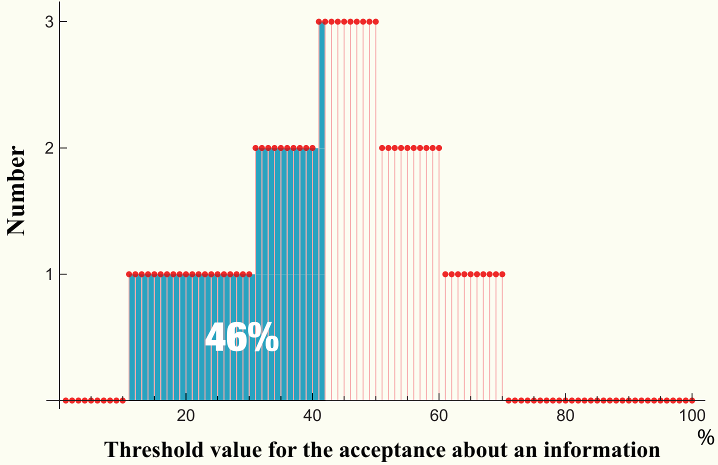
Granovetter's threshold model

A threshold distribution



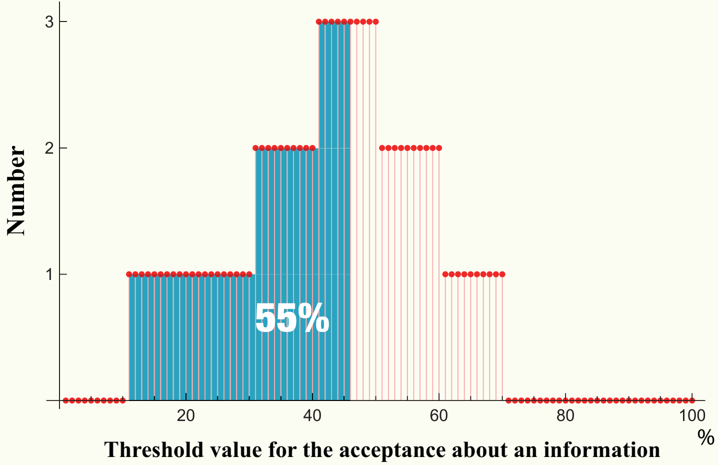
Granovetter's threshold model

A threshold distribution



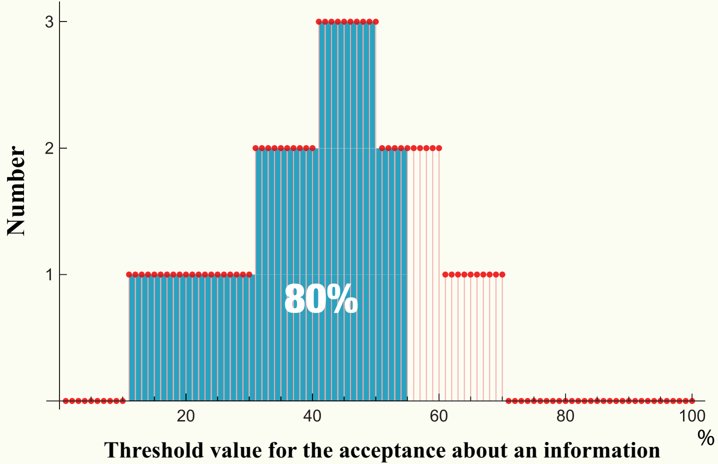
Granovetter's threshold model

A threshold distribution



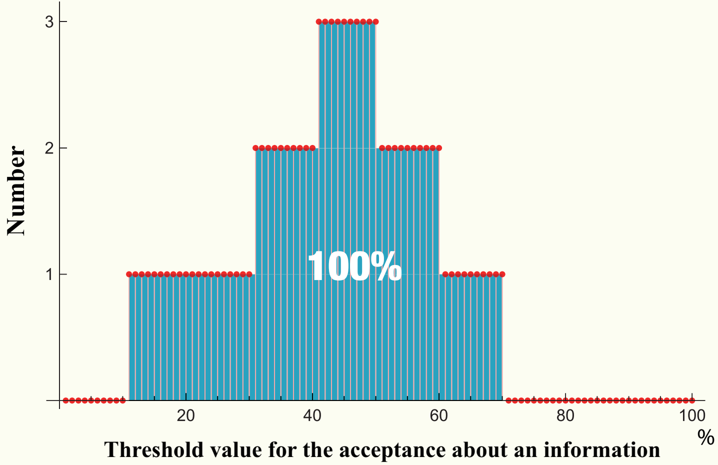
Granovetter's threshold model

A threshold distribution



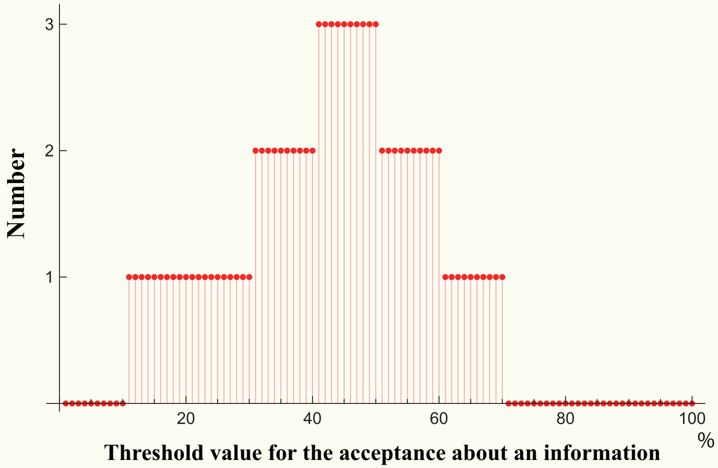
Granovetter's threshold model

A threshold distribution



Granovetter's threshold model

A threshold distribution



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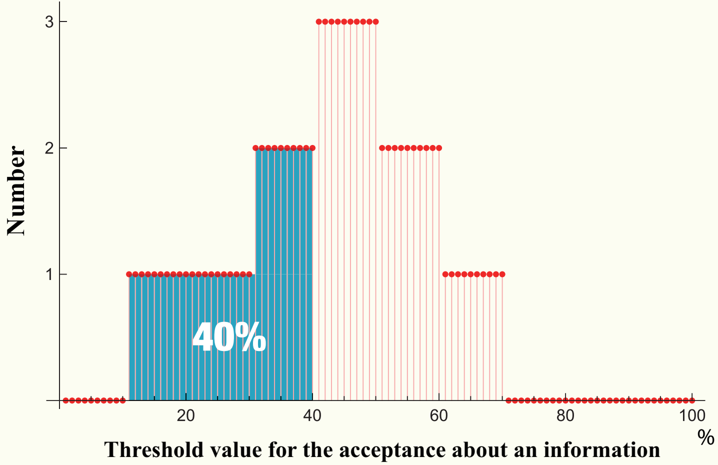

Granovetter's threshold model

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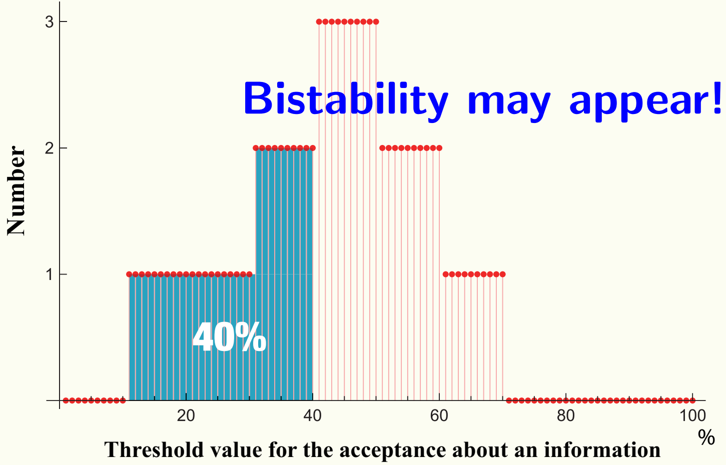


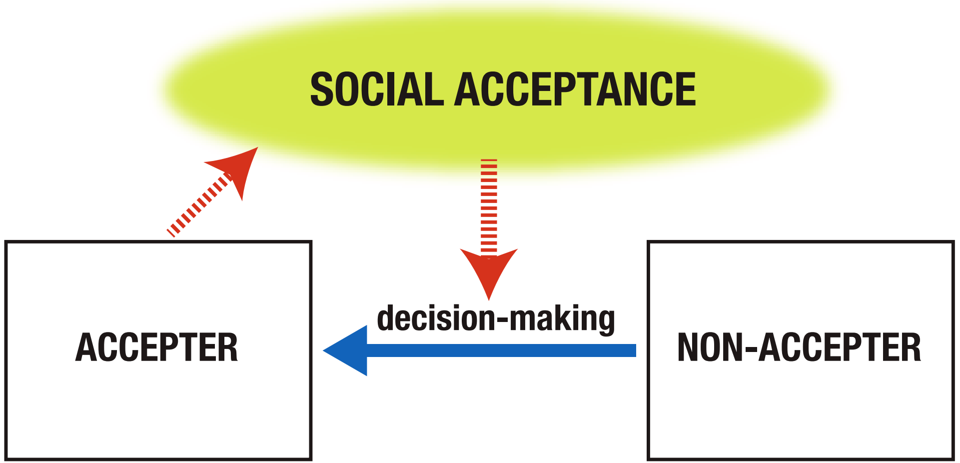
Unclear correspondence to the temporal variation in the number of accepters.

$$\Downarrow$$

What population dynamics model is reasonable?

How does the heterogeneity of individuality influence the information spread within a population?

Population dynamics modeling for information spread



Population dynamics modeling for information spread

Rule for the decision-making on accepting the information

$$\begin{cases} \xi \leq \alpha P & \Longrightarrow \text{The information may be accepted.} \\ \xi > \alpha P & \Longrightarrow \text{The information is not accepted with the denial/disregard.} \end{cases}$$

- ξ : Threshold value of individual.
- P : Accepter frequency within the community.
- αP : Strength of social acceptance regarding the information, now assumed to be proportional to the accepter frequency.

Population dynamics modeling for information spread

Threshold distribution

$$F(x) = \textbf{Prob}(\xi \leq x) = \int_{-\infty}^{x} f(\xi)\, d\xi$$

$$\lim_{x \to -\infty} F(x) = 0; \quad \lim_{x \to \infty} F(x) = 1; \quad f(\xi) \geq 0;$$

$$\bar{\xi} = \int_{-\infty}^{\infty} \xi f(\xi)\, d\xi; \quad \sigma^2 = \int_{-\infty}^{\infty} (\xi - \bar{\xi})^2 f(\xi)\, d\xi$$

Population dynamics modeling for information spread

Threshold distribution

$$F(x) = \textbf{Prob}(\xi \le x) = \int_{-\infty}^{x} f(\xi)\, d\xi$$

Mathematical assumption

$$F(x) = \begin{cases} 0 & \text{for } x \le 0; \\ 1 & \text{for } x \ge \alpha; \end{cases} \qquad f(\xi) = \begin{cases} 0 & \text{for } \xi \le 0; \\ \text{non-negative} & \text{for } \xi \in (0, \alpha); \\ 0 & \text{for } \xi \ge \alpha. \end{cases}$$

Initial accepter frequency

$$P_0 = \int_{-\infty}^{\infty} \varphi_0(\xi) f(\xi)\, d\xi = \int_0^{\alpha} \varphi_0(\xi) f(\xi)\, d\xi$$

$$0 \leq \varphi_0(\xi) \leq 1; \quad 0 < P_0 \leq \int_{-\infty}^{\infty} f(\xi)\, d\xi = \int_0^{\alpha} f(\xi)\, d\xi = 1$$

NOTE: They are excepted from the rule of decision-making!

Population dynamics modeling for information spread

Initial accepters whose thresholds are BEYOND αP_t at time t

$$N \int_{\alpha P_t}^{\alpha} \varphi_0(\xi) f(\xi)\, d\xi$$

- N : Number of individuals in the community.
- P_t : Accepter frequency at time t.
- αP_t : Strength of social acceptance at time t.

Population dynamics modeling for information spread

Initial accepters whose thresholds are BEYOND αP_t at time t

$$N \int_{\alpha P_t}^{\alpha} \varphi_0(\xi) f(\xi)\, d\xi$$

Individuals with the threshold BELOW αP_t at time t, except for the initial accepters

$$N \int_{0}^{\alpha P_t} \{1 - \varphi_0(\xi)\} f(\xi)\, d\xi$$

Population dynamics modeling for information spread

Accepters with the threshold BELOW αP_t at time t except for the initial accepters

$$N(P_t - P_0)$$

Non-accepters with the threshold BELOW αP_t at time t

$$N\int_0^{\alpha P_t} \{1-\varphi_0(\xi)\} f(\xi)\, d\xi - N(P_t - P_0)$$

$$= NF(\alpha P_t) + N\int_{\alpha P_t}^{\alpha} \varphi_0(\xi) f(\xi)\, d\xi - NP_t$$

Population dynamics modeling for information spread

Non-accepters with the threshold BELOW αP_t at time t

$$NF(\alpha P_t) + N\int_{\alpha P_t}^{\alpha} \varphi_0(\xi) f(\xi)\, d\xi - NP_t$$

New accepters in unit time step

$$NP_{t+1} - NP_t = \gamma B(P_t) \left\{ NF(\alpha P_t) + N\int_{\alpha P_t}^{\alpha} \varphi_0(\xi) f(\xi)\, d\xi - NP_t \right\}$$

$B(P)$: **Probability** of having the opportunity to make a decision under the given accepter frequency P, with $B(0) = 0$.

γ : **Probability** of making the decision to accept the information per opportunity.

Recurrence relation for the temporal sequence of accepter frequencies

$$P_{t+1} = \{1 - \gamma B(P_t)\} P_t + \gamma B(P_t) \left\{ F(\alpha P_t) + \int_{\alpha P_t}^{\alpha} \varphi_0(\xi) f(\xi)\, d\xi \right\}$$

$$P_0 := \int_{-\infty}^{\infty} \varphi_0(\xi) f(\xi)\, d\xi = \int_0^{\alpha} \varphi_0(\xi) f(\xi)\, d\xi$$

Population dynamics modeling for information spread

Recurrence relation for the temporal sequence of accepter frequencies

$$P_{t+1} = \{1 - \gamma B(P_t)\} P_t + \gamma B(P_t) \left\{ F(\alpha P_t) + \int_{\alpha P_t}^{\alpha} \varphi_0(\xi) f(\xi)\, d\xi \right\}$$

Theorem

The sequence $\{P_t\}$ monotonically increases and converges to a value $P^* \in [P_0, 1]$ for any $P_0 \in (0,1)$ as $t \to \infty$.

With

the frequency-proportional probability $B(P_t) = bP_t$ $(0 < b \leq 1)$, and uniformly chosen initial accepters with $\varphi_0(\xi) = \varphi_0$,

Recurrence relation on the temporal sequence of accepter frequency

$$P_{t+1} = \Big[1 + \gamma b\big\{\varphi_0 - P_t + (1-\varphi_0)F(\alpha P_t)\big\}\Big]P_t$$

$$P_0 := \int_{-\infty}^{\infty} \varphi_0(\xi) f(\xi)\, d\xi = \int_{-\infty}^{\infty} \varphi_0 f(\xi)\, d\xi = \varphi_0 \int_{-\infty}^{\infty} f(\xi)\, d\xi = \varphi_0$$

Population dynamics modeling for information spread

Recurrence relation on the temporal sequence of accepter frequency

$$P_{t+1} = \Big[1 + \gamma b\big\{\varphi_0 - P_t + (1-\varphi_0)F(\alpha P_t)\big\}\Big]P_t$$

Lemma (No spread)

If $P_0 = \varphi_0 \leq \theta_{\inf} := \xi_{\inf}/\alpha$, the accepter frequency remains the initial frequency P_0 with no increase at any time step. Otherwise, it temporally increases.

Lemma (Complete spread)

If $P_0 = \varphi_0 \geq \theta_{\sup} := \xi_{\sup}/\alpha$, the accepter frequency monotonically increases toward 1.

How does the heterogeneity of individuality influence the information spread within a population?

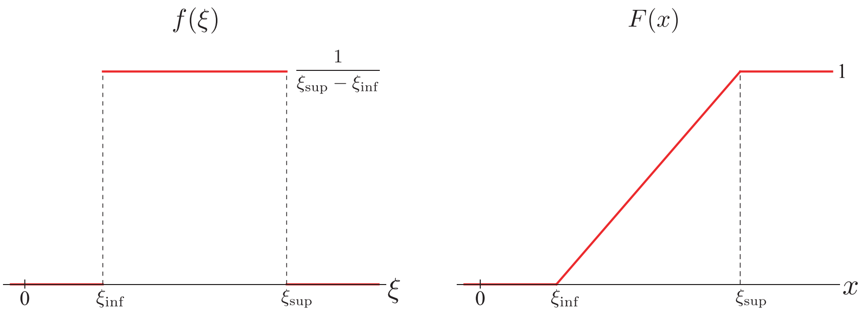
∼ How does the dynamical nature of information spread depend on the threshold distribution defined by $f(\xi)$?

Population dynamics model for information spread

Recurrence relation on the temporal sequence of accepter frequency

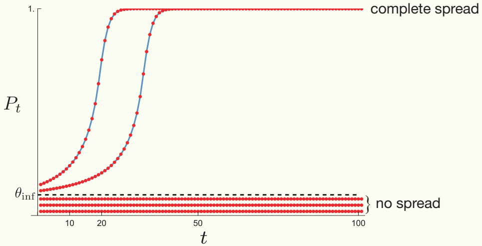
$$P_{t+1} = \left[1 + \gamma b\left\{\varphi_0 - P_t + (1-\varphi_0)F(\alpha P_t)\right\}\right] P_t$$

Uniform distribution



Population dynamics model for information spread

For **the uniform threshold distribution**, the accepter frequency P_t monotonically approaches 1 as time passes if $P_0 = \varphi_0 > \theta_{\text{inf}}$, while it remains the initial frequency, $P_t \equiv \varphi_0$, if $\varphi_0 \leq \theta_{\text{inf}}$.



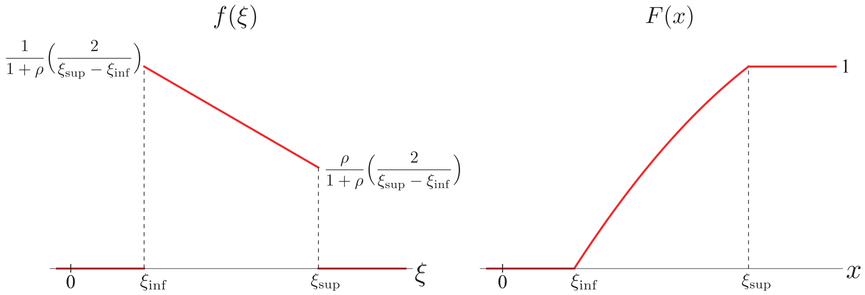
$P_0 = \varphi_0 = 0.02, 0.05, 0.08, 0.12, 0.15;\ \alpha = 1.0;\ \gamma b = 0.5;\ \xi_{\text{inf}} = 0.1;\ \xi_{\text{sup}} = 0.6;\ \theta_{\text{inf}} := \xi_{\text{inf}}/\alpha = 0.1.$

Population dynamics model for information spread

Recurrence relation on the temporal sequence of accepter frequency

$$P_{t+1} = \left[1 + \gamma b\left\{\varphi_0 - P_t + (1-\varphi_0)F(\alpha P_t)\right\}\right] P_t$$

A linear example of monotonically decreasing distribution



Population dynamics model for information spread

For the monotonically decreasing threshold distribution, the accepter frequency P_t monotonically approaches 1 as time passes for the initial accepter frequency such that $P_0 = \varphi_0 > \theta_{\mathrm{inf}}$, while it remains the initial frequency, $P_t \equiv \varphi_0$, for $\varphi_0 \leq \theta_{\mathrm{inf}}$.

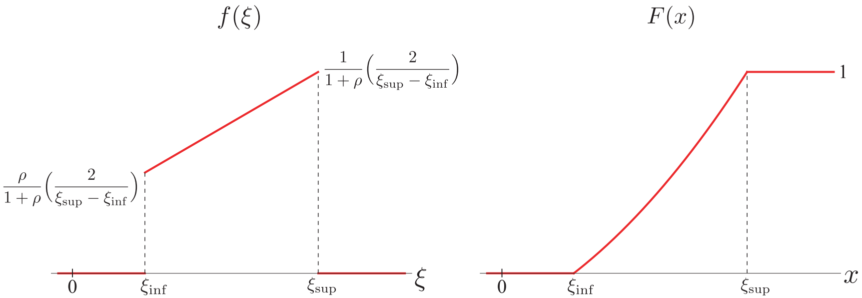
Same as for the uniform threshold distribution!

Population dynamics model for information spread

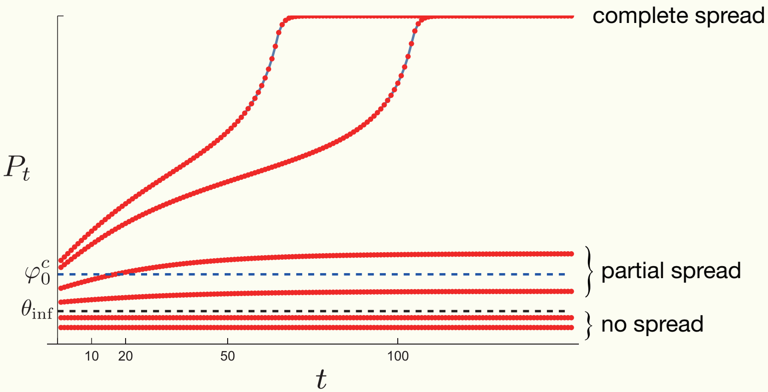
Recurrence relation on the temporal sequence of accepter frequency

$$P_{t+1} = \left[1 + \gamma b \left\{\varphi_0 - P_t + (1-\varphi_0) F\left(\alpha P_t\right)\right\}\right] P_t$$

A linear example of monotonically increasing distribution



Population dynamics model for information spread



$P_0 = \varphi_0 = 0.05, 0.08, 0.127, 0.170, 0.233, 0.255$; $\alpha = 1.0$; $\gamma b = 0.5$; $\xi_{\text{inf}} = \theta_{\text{inf}} = 0.1$; $\xi_{\text{sup}} = 0.9$; $\rho = 0.3$; $\varphi_0^c = 0.212$; $\rho_c = 0.8$.

A partial spread of information occurs only when the increasing threshold distribution is sufficiently biased towards the higher value.

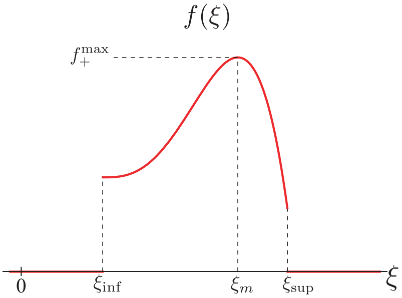
In a population with a high mean threshold value, and therefore a sufficiently conservative characteristic according to the acceptance of the spreading information, only a partial spread may occur, even if the spread is ongoing.

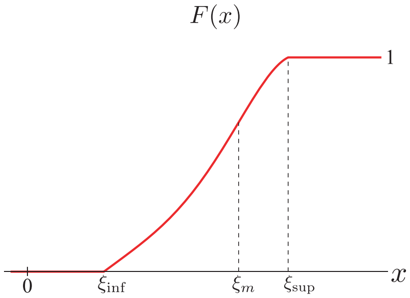
Population dynamics model for information spread

Recurrence relation on the temporal sequence of accepter frequency

$$P_{t+1} = \left[1 + \gamma b \left\{\varphi_0 - P_t + (1-\varphi_0) F(\alpha P_t)\right\}\right] P_t$$

A schematic example of unimodal threshold distribution





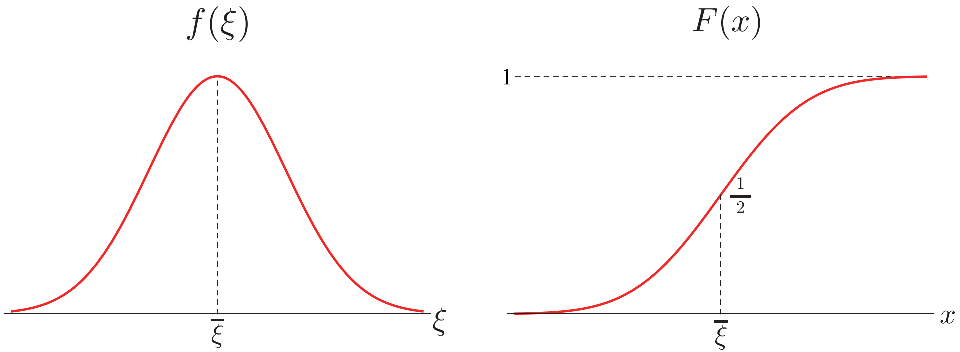
The behavioral characteristics of the accepter frequency is qualitatively same as for the monotonically increasing distribution.

It is implied that the complete spread of an information becomes harder as the unimodal threshold distribution is biased towards the larger value.

For the unimodal threshold distribution that is biased towards the larger value, the complete spread of an information becomes more challenging, while the partial spread becomes relatively more successful when it occurs.

Population dynamics model for information spread

An extra case: Normal threshold distribution

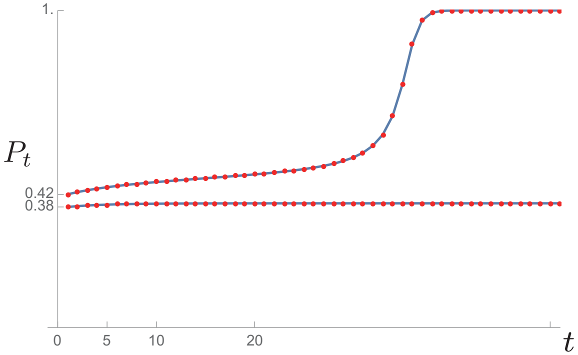


$$f(\xi) := \frac{1}{\sigma\sqrt{2\pi}} \mathrm{e}^{-(\xi-\bar{\xi})^2/2\sigma^2};$$

$$F(\alpha P_t) = \int_{-\infty}^{\alpha P_t} f(\xi)\, d\xi = \frac{1}{\sqrt{\pi}} \int_{-\infty}^{(P_t-\bar{\xi}/\alpha)/(\sigma\sqrt{2}/\alpha)} \mathrm{e}^{-x^2}\, dx$$

Population dynamics model for information spread

An extra case: Normal threshold distribution



$P_0 = \varphi_0 = 0.38, 0.42;\ \gamma b = 0.8;\ \bar{\xi}/\alpha = 0.6;\ \sigma/\alpha = 0.1.$

Qualitatively same as the results for the general unimodal threshold distribution.

Population dynamics model for information spread

For a spreading information to be widely accepted, the threshold distribution must have a sufficiently small mean and a large variance within the community.

Such a community can be considered one that has a high diversity of individuality and readily accepts the information.

In contrast, for a community with a large mean and a small variance in the threshold distribution, meaning many cautious members are hesitant to accept the information, its spread would be unsuccessful.

Population dynamics model for information spread

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Negative aspect of information spread

Diffusion of an innovative technology could change people's lifestyle.

May bring problems to our societies,
like environmental pollution, global warming, drug disasters, etc.

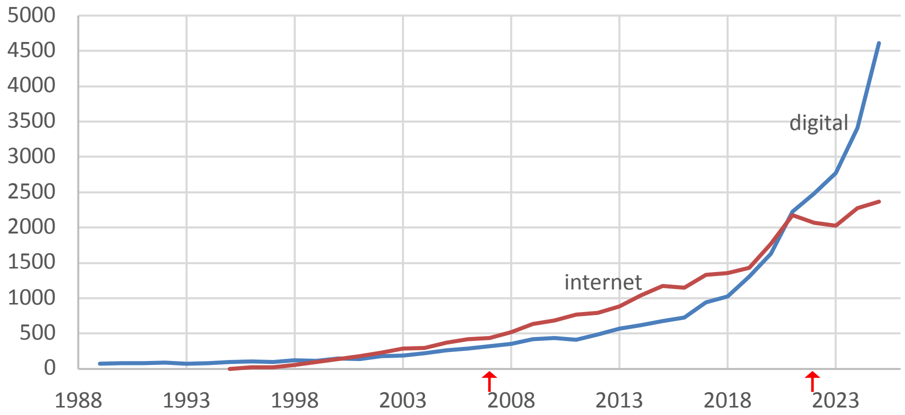
Over the past decade, similar scenarios have emerged and become increasingly serious across various aspects of digital technologies.

e.g., digital dependency, smartphone dependency, internet dependency, internet gaming disorder, SNS dependency, AI dependency.

$\sim$ online behavioral addiction

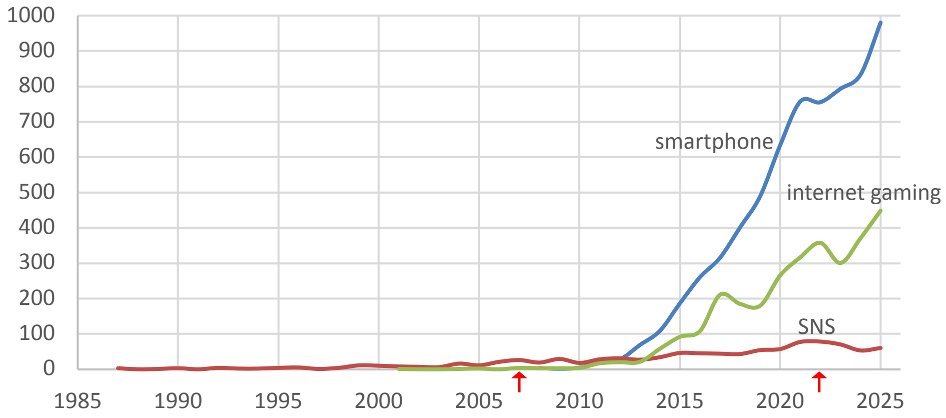
Online behavioral addiction

Hit counts of Scopus search with “dependency”, “addiction”, or “disorder”

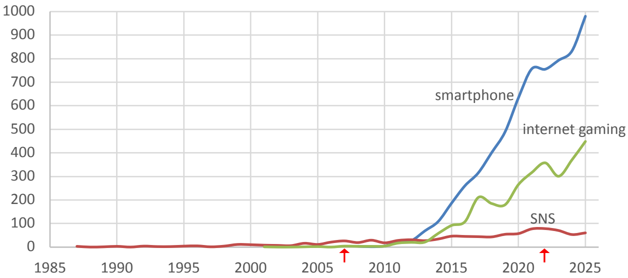


Online behavioral addiction

Hit counts of Scopus search with “dependency”, “addiction”, or “disorder”



Internet gaming disorder



Online gaming disorder

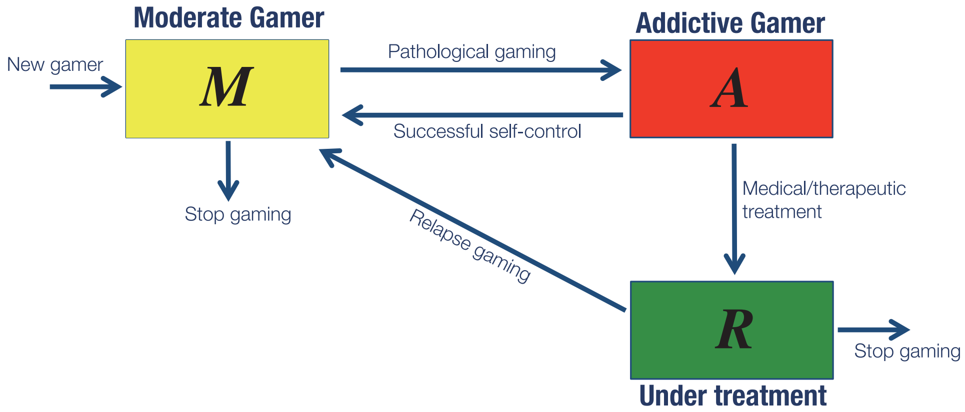
Online gaming disorder (IGD)

"Gaming disorder, predominantly online"

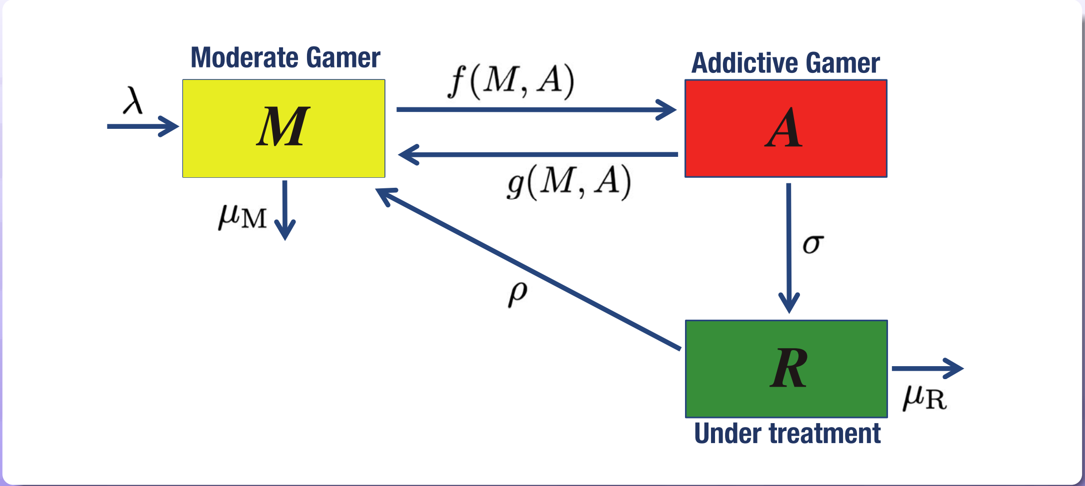
A pattern of gaming behavior ("digital-gaming" or "video-gaming") characterized by impaired control over gaming, increasing priority given to gaming over other activities to the extent that gaming takes precedence over other interests and daily activities, and continuation or escalation of gaming despite the occurrence of negative consequences.

(WHO, ICD-11, 2018. https://www.who.int/standards/classifications/frequently-asked-questions/gaming-disorder)

Population dynamics model for online gaming disorder



Population dynamics model for online gaming disorder



- Both of $f(x,y)$ and $g(x,y)$ are positive and conituous on $[0,\infty) \times [0,\infty)$;
- $f(x,y)$ is increasing in terms of x and y;
- $g(x,y)$ is decreasing in terms of x and y.

Population dynamics model for online gaming disorder

Genetic model

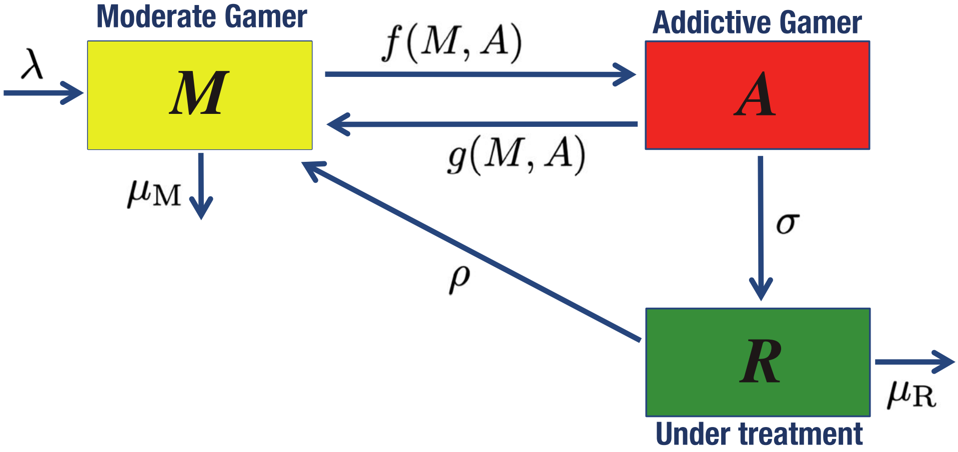
$$\frac{dM}{dt} = \lambda - f(M,A)M + g(M,A)A - \mu_{\mathrm{M}}M + \rho R$$

$$\frac{dA}{dt} = f(M,A)M - g(M,A)A - \sigma A$$

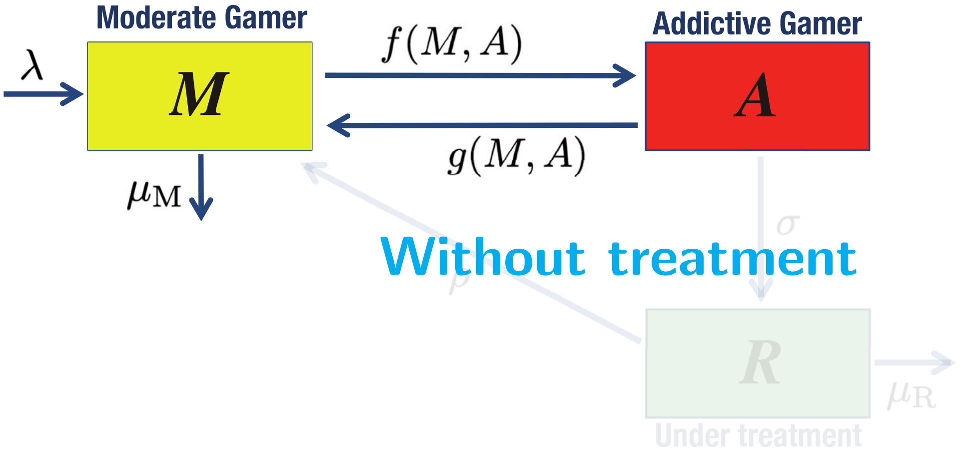
$$\frac{dR}{dt} = \sigma A - \rho R - \mu_{\mathrm{R}}R$$

- **Both of $f(x,y)$ and $g(x,y)$ are positive and conituous on $[0,\infty) \times [0,\infty)$;**
- **$f(x,y)$ is increasing in terms of x and y;**
- **$g(x,y)$ is decreasing in terms of x and y.**

Population dynamics model for online gaming disorder



Population dynamics model for online gaming disorder



Without treatment

Population dynamics model for online gaming disorder

Model without treatment

$$\frac{dM}{dt} = \lambda - f(M,A)M + g(M,A)A - \mu_{\mathrm{M}} M + \rho R$$

$$\frac{dA}{dt} = f(M,A)M - g(M,A)A - \sigma A$$

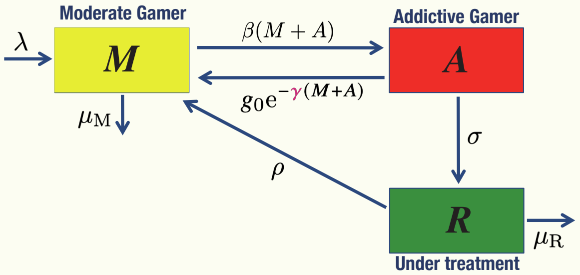
$$\frac{dR}{dt} = \sigma A - \rho R - \mu_{\mathrm{R}} R$$

- Both of $f(x,y)$ and $g(x,y)$ are positive and conituous on $[0,\infty) \times [0,\infty)$;
- $f(x,y)$ is increasing in terms of x and y;
- $g(x,y)$ is decreasing in terms of x and y.

Population dynamics model for online gaming disorder

A simple model

$$f(M, A) = \beta(M + A); \quad g(M, A) = g_0\, \mathrm{e}^{-\gamma(M+A)}$$



β: Coefficient of transition to addiction;

g_0: Intrinsic self-recovery rate from addiction;

γ: Coefficient of the bond of online social relationship

Population dynamics model without treatment

A simple model WITHOUT treatment

$$\frac{dM}{dt} = \lambda - \beta(M + A)M + g_0\, e^{-\gamma(M+A)} A - \mu_M M + \rho R$$

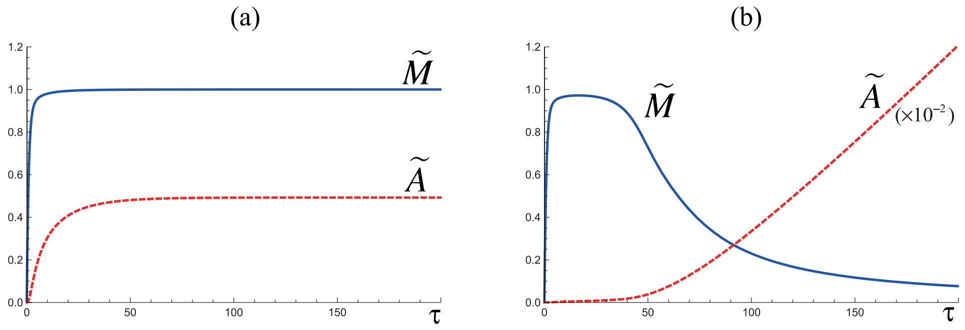
$$\frac{dA}{dt} = \beta(M + A)M - g_0\, e^{-\gamma(M+A)} A - \sigma A$$

$$\frac{dR}{dt} = \sigma A - \rho R - \mu_R R$$

- β: Coefficient of transition to addiction;
- g_0: Intrinsic self-recovery rate from addiction;
- γ: Coefficient of the bond of online social relationship

Population dynamics model for online gaming disorder

Temporal variations for the simple model WITHOUT treatment



Numerically obtained temporal variations of $(\widetilde{M}, \widetilde{A}) = (M/(\lambda/\mu_{\mathrm{M}}), A/(\lambda/\mu_{\mathrm{M}}))$: (a) $\widetilde{\gamma} = 0.8$; (b) $\widetilde{\gamma} = 1.0$. Commonly, $\tau = \mu_{\mathrm{M}} t$; $(\widetilde{M}(0), \widetilde{A}(0)) = (0.0, 0.0)$; $\widetilde{\beta} = 0.1$; $\widetilde{g}_0 = 1.0$; $\widetilde{\gamma}_c = 0.832529$.

Population dynamics model for online gaming disorder

A simple model WITH treatment

$$\frac{dM}{dt} = \lambda - \beta(M+A)M + g_0\,\mathrm{e}^{-\gamma(M+A)}A - \mu_{\mathrm{M}}M + \rho R$$

$$\frac{dA}{dt} = \beta(M+A)M - g_0\,\mathrm{e}^{-\gamma(M+A)}A - \sigma A$$

$$\frac{dR}{dt} = \sigma A - \rho R - \mu_{\mathrm{R}}R$$

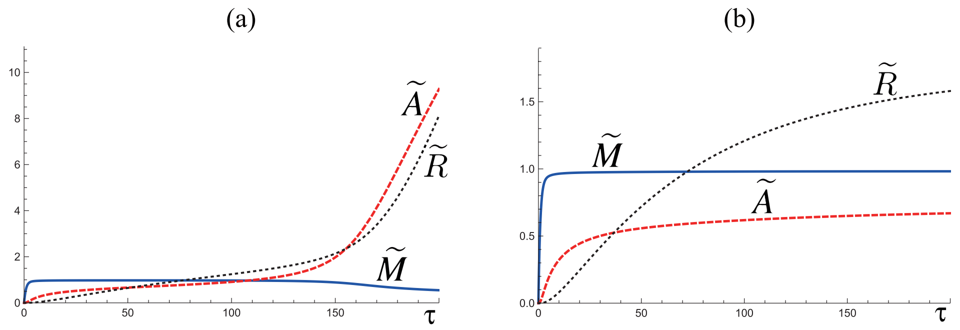
β: **Coefficient of transition to addiction;**

g_0: **Intrinsic self-recovery rate from addiction;**

γ: **Coefficient of the bond of online social relationship**

Population dynamics model for online gaming disorder

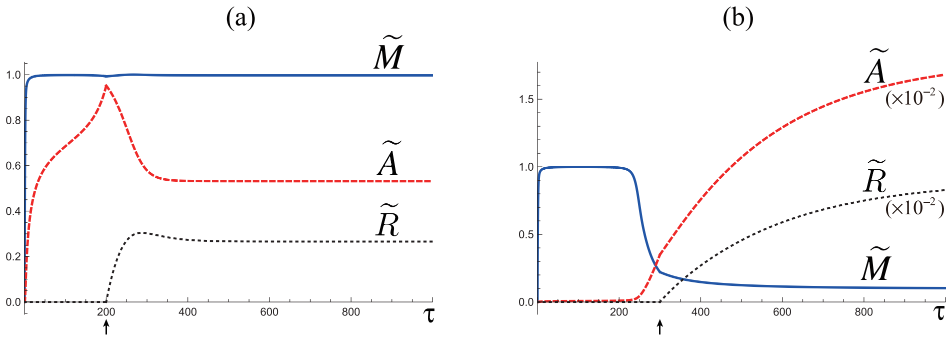
Temporal variations for the simple model WITH treatment



Numerically obtained temporal variations of $(\widetilde{M}, \widetilde{A}, \widetilde{R}) = (M/(\lambda/\mu_{\rm M}), A/(\lambda/\mu_{\rm M}), R/(\lambda/\mu_{\rm M}))$**:** $\widetilde{\sigma} = \sigma/\mu_{\rm M} =$ **(a) 0.04; (b) 0.05. Commonly,** $\tau = \mu_{\rm M} t$; $(\widetilde{M}(0), \widetilde{A}(0), \widetilde{R}(0)) = (0.0, 0.0, 0.0)$; $\widetilde{\beta} = (\lambda/\mu_{\rm M})\beta/\mu_{\rm M} = 0.1$; $\widetilde{g}_0 = g_0/\mu_{\rm M} = 1.0$; $\widetilde{\gamma} = (\lambda/\mu_{\rm M})\gamma = 1.0$; $\widetilde{\rho} = \rho/\mu_{\rm M} = 0.01$; $\widetilde{\mu} = \mu_{\rm R}/\mu_{\rm M} = 0.01$; $\widetilde{\gamma}_c = 0.832529$; $\widetilde{\sigma}_c = 0.0497871$.

Population dynamics model for online gaming disorder

Temporal variations for the simple model "WITHOUT → WITH" treatment



Numerically obtained temporal variations of $(\widetilde{M}, \widetilde{A}, \widetilde{R}) = (M/(\lambda/\mu_M), A/(\lambda/\mu_M), R/(\lambda/\mu_M))$ with a moment $\tau = \mu_M t = \tau_1$ before which there is no treatment and at which the treatment starts with $\widetilde{\sigma} = 0.01$: $\tau_1 =$ (a) 200; (b) 300. Commonly, $(\widetilde{M}(0), \widetilde{A}(0), \widetilde{R}(0)) = (0.0, 0.0, 0.0)$; $\widetilde{\beta} = (\lambda/\mu_M)\beta/\mu_M = 0.1$; $\widetilde{g}_0 = g_0/\mu_M = 1.0$; $\widetilde{\gamma} = (\lambda/\mu_M)\gamma = 0.84$; $\widetilde{\rho} = \rho/\mu_M = 0.01$; $\widetilde{\mu} = \mu_R/\mu_M = 0.01$; ; $\alpha_c = 0.704641$; $\widetilde{\gamma}_c = 0.832529$.

Population dynamics model for online gaming disorder

When the bond of online social relationships is weak, the population of addictive gamers remains relatively low. However, once the bond becomes sufficiently strong, the number of addictive gamers can show a sudden surge.

Only a treatment operation with a sufficiently high efficiency would be successful in suppressing the population size of addictive gamers at a certain level.

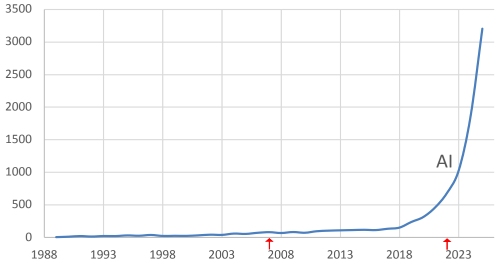
The effectiveness of a treatment operation heavily relies on the inherent social reinforcements provided by online gaming. The stronger social reinforcements make the treatment operation more challenging.

The outcome of a treatment operation could significantly depend on the timing of its commencement.

REFERENCE: Seno, H., 2021. A mathematical model of population dynamics about the internet gaming addiction, *Nonlinear Analysis: Modelling and Control* **26**(5): 861–883.

ON THE OTHER HAND, TODAY

we have already been aware of the emergence of a new type of internet addiction stemming from the dependency on generative AI.



Online behavioral addiction

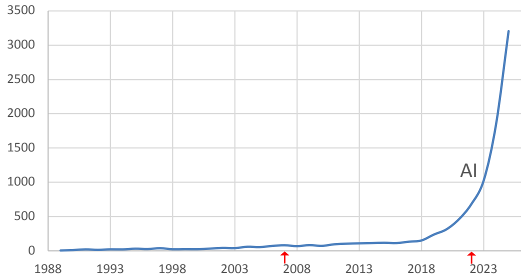
ON THE OTHER HAND, TODAY

we have already been aware of the emergence of a new type of internet addiction stemming from the dependency on generative AI.

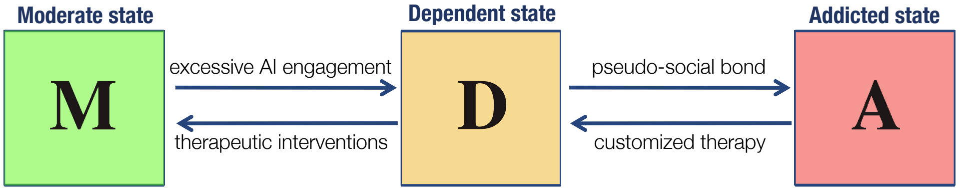
AI dependency has a distinct nature compared to online gaming disorders, especially in terms of its population dynamics. It is triggered by a pseudo-social bond with a virtual personal relationship created by AI, which is actually independent of the other users.

We should recognize its high potential to spread widely across our society, while acknowledging that AI usage is certainly beneficial for a variety of social activities.

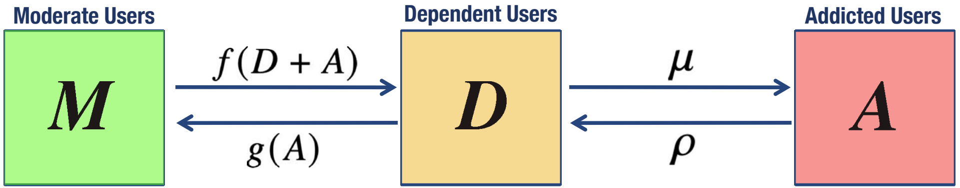
AI dependency



Population dynamics model for AI dependency



Population dynamics model for AI dependency



f: Transition rate from moderate to dependent state;
g: Transition rate from dependent to moderate state;

Both of $f(x)$ and $g(x)$ are positive, differentiable, and monotonically increasing for $x > 0$, satisfying that $f(0) = f_0 > 0$ and $g(0) = g_0 > 0$.

μ: Transition rate from dependent to addicted state;
ρ: Transition rate from addicted to dependent state.

$M(t) + D(t) + A(t) = N$ (a positive constant) for any $t \geq 0$.

Population dynamics model for AI dependency

Genetic model

$$\frac{dM}{dt} = -f(D+A)M + g(A)D$$
$$\frac{dD}{dt} = \quad f(D+A)M - g(A)D - \mu D + \rho A$$
$$\frac{dA}{dt} = \mu D - \rho A$$

f: **Transition rate from moderate to dependent state;**
g: **Transition rate from dependent to moderate state;**

Both of $f(x)$ and $g(x)$ are positive, differentiable, and monotonically increasing for $x > 0$, satisfying that $f(0) = f_0 > 0$ and $g(0) = g_0 > 0$.

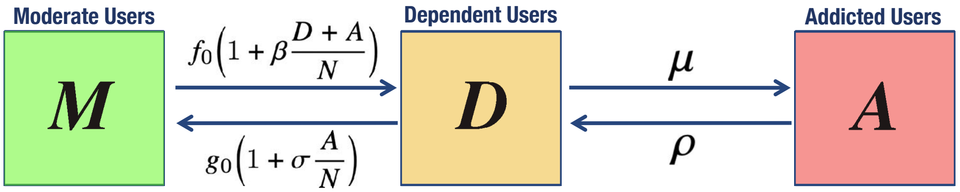
μ: **Transition rate from dependent to addicted state;**
ρ: **Transition rate from addicted to dependent state.**

$M(t) + D(t) + A(t) = N$ **(a positive constant) for any $t \geq 0$.**

Population dynamics model for AI dependency

A simple model

$$f(D+A) = f_0\left(1+\beta\frac{D+A}{N}\right); \quad g(A) = g_0\left(1+\sigma\frac{A}{N}\right)$$



- f_0: Intrinsic likelihood to become dependent on AI;
- β: Coefficient of enhancement to AI dependency;
- g_0: User's inherent tendency to overcome AI dependency;
- σ: Coefficient of enhancement to the recovery from AI dependency.

Population dynamics model for AI dependency

A simple model

$$\frac{dM}{dt} = -f_0\left(1 + \beta\frac{D+A}{N}\right)M + g_0\left(1 + \sigma\frac{A}{N}\right)D$$

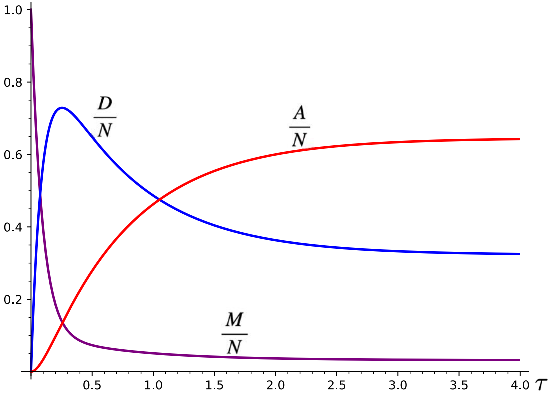
$$\frac{dD}{dt} = \quad f_0\left(1 + \beta\frac{D+A}{N}\right)M - g_0\left(1 + \sigma\frac{A}{N}\right)D - \mu D + \rho A$$

$$\frac{dA}{dt} = \mu D - \rho A$$

f_0: Intrinsic likelihood to become dependent on AI;
β: Coefficient of enhancement to AI dependency;
g_0: User's inherent tendency to overcome AI dependency;
σ: Coefficient of enhancement to the recovery from AI dependency.

Population dynamics model for AI dependency

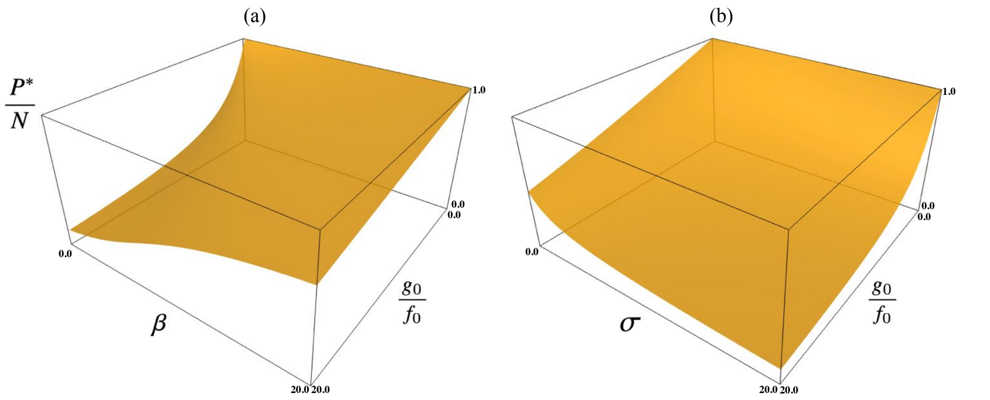
Temporal variation for the simple model



Temporal variation of $\left(\frac{M}{N}, \frac{D}{N}, \frac{A}{N}\right)$. Numerically drawn with $\tau = \mu t$; $\phi_0 = f_0/\mu = 10.0$; $\beta = 0.01$; $r = \rho/\mu = 0.5$; $\gamma_0 = g_0/\mu = 1.0$; $\sigma = 0.01$; $\left(\frac{M(0)}{N}, \frac{D(0)}{N}, \frac{A(0)}{N}\right) = (1.0, 0.0, 0.0)$.

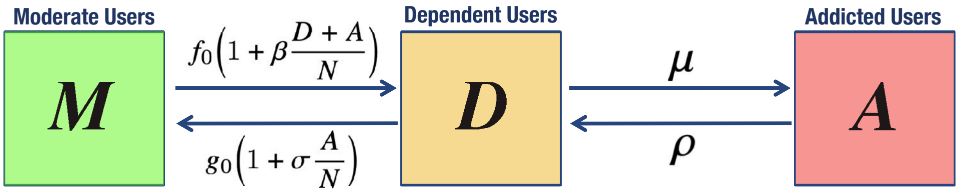
Population dynamics model for AI dependency

$(\beta, \sigma, \frac{g_0}{f_0})$-dependence of $\frac{D^*+A^*}{N}$ for the simple model



Numerically drawn with $r = \rho/\mu = 0.5$, and (a) $\sigma = 0.0$; (b) $\beta = 10.0$.

Population dynamics model for AI dependency

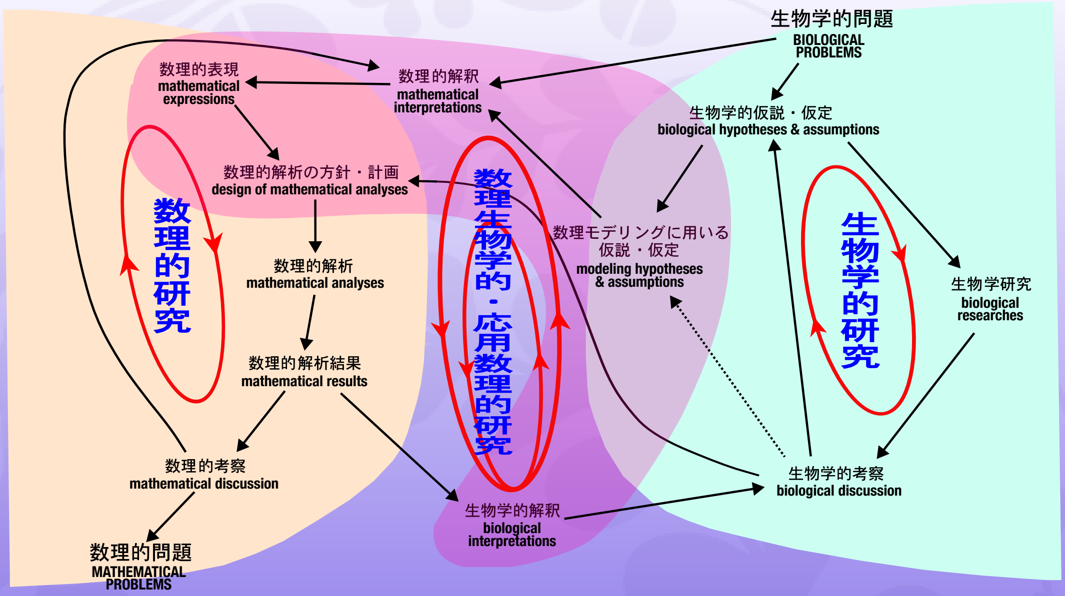


Given the strong attraction to AI usage, it is imperative that we conduct researches and make preparations to mitigate the rise in AI dependency.

REFERENCE: Seno, H., 2026. A population dynamics model for the rampancy of AI dependency, **in preparation**.

Epilogue

Epilogue



生物学的問題
BIOLOGICAL PROBLEMS
数理的表現
mathematical expressions
数理的解釈
mathematical interpretations
生物学的仮説・仮定
biological hypotheses & assumptions
数理的解析の方針・計画
design of mathematical analyses
数理モデリングに用いる仮説・仮定
modeling hypotheses & assumptions
数理的研究
数理生物学的・応用数理的研究
生物学的研究
数理的解析
mathematical analyses
生物学研究
biological researches
数理的解析結果
mathematical results
数理的考察
mathematical discussion
生物学的考察
biological discussion
生物学的解釈
biological interpretations
数理的問題
MATHEMATICAL PROBLEMS

Distinct four logistic equations

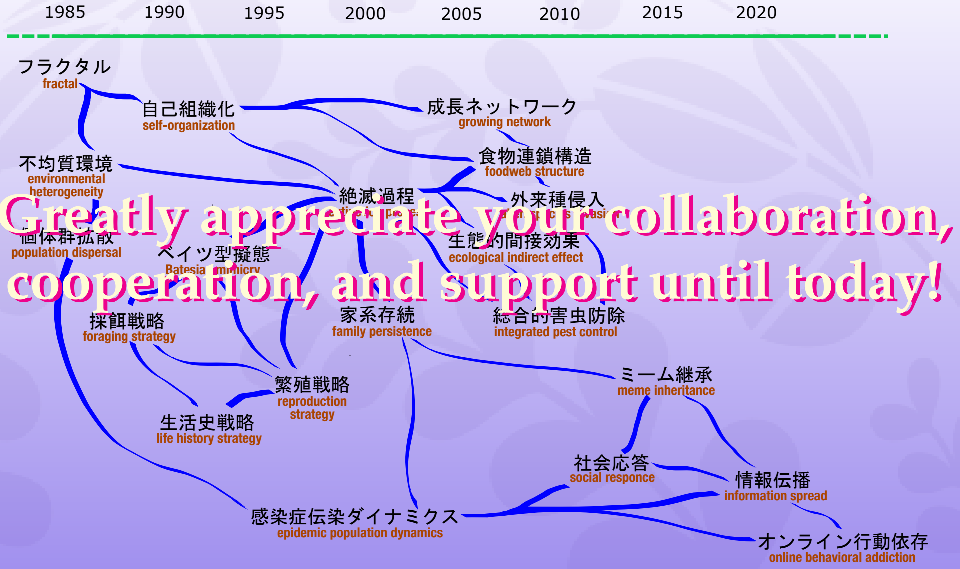
[L-1] $$\frac{dN(t)}{dt} = \{r_0 - \beta N(t)\}N(t)$$

[L-2] $$\frac{dN(t)}{dt} = r_0\left\{1 - \frac{N(t)}{K}\right\}N(t)$$

[L-3] $$\frac{dN(t)}{dt} = r_0 N(t) - b\{N(t)\}^2$$

[L-4] $$\frac{dN(t)}{dt} = \{r_0 - \beta N(t)\}N(t) - b\{N(t)\}^2$$

Epilogue



Greatly appreciate your collaboration, cooperation, and support until today!