感染症伝染ダイナミクスの数理モデル初歩

A First Step into Mathematical Model on The Epidemic Dynamics of Infectious Disease

瀬野裕美

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Outline

Prologue: Epidemic dynamics of infectious disease

Epidemic dynamics model

- Generic SIR model
- Kermack–McKendrick SIR model
- Basic reprodution number $\mathscr{R}_0$
- Kermack–McKendrick SIRS model
- Force of infection

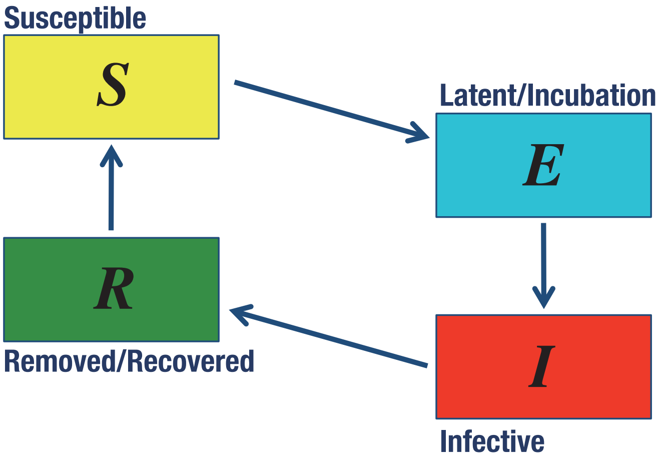
Epilogue: Various factors on the epidemic dynamics

Prologue:

Epidemic dynamics of infectious disease

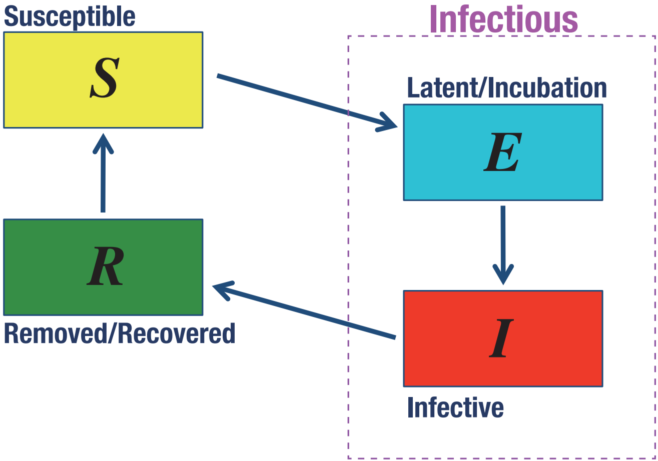
Epidemic dynamics of infectious disease

$S \longrightarrow E \longrightarrow I \longrightarrow R \longrightarrow S$



Epidemic dynamics of infectious disease

$S \to E \to I \to R \to S$



Epidemic dynamics of infectious disease

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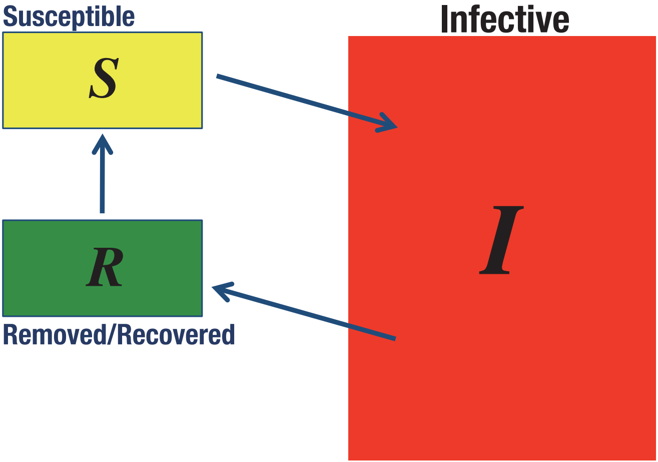

Epidemic dynamics of infectious disease

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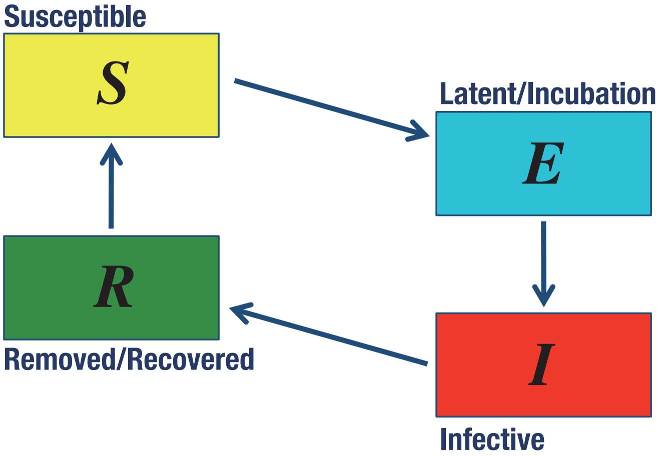

Epidemic dynamics of infectious disease

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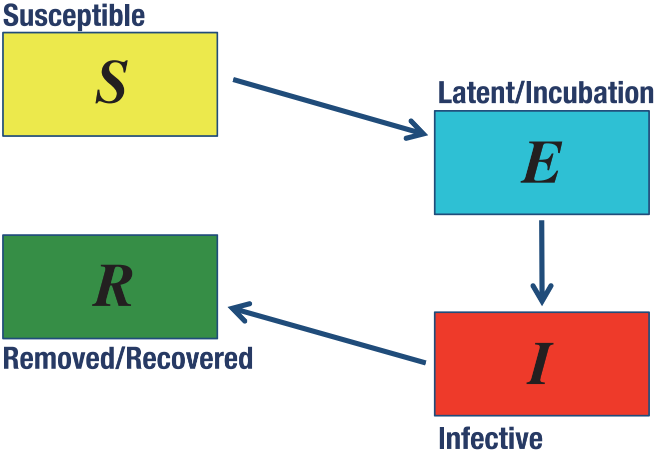
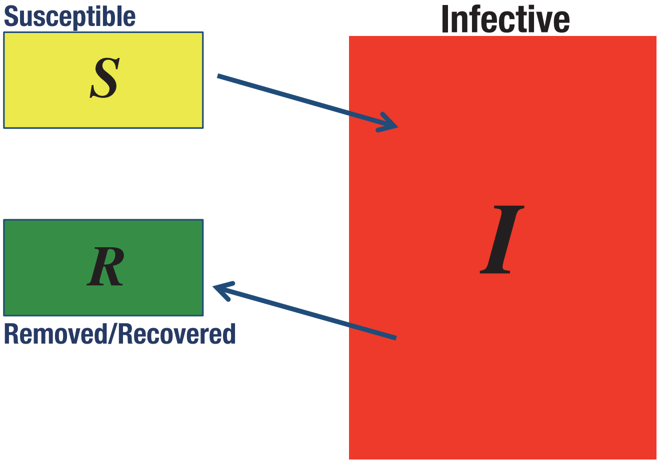


Epidemic dynamics of infectious disease

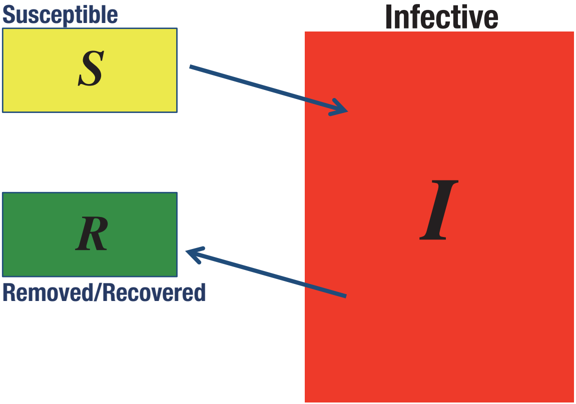
$S \longrightarrow I \longrightarrow R$



Epidemic dynamics model

Generic SIR model

Generic SIR model



Generic SIR model

$$\frac{dS(t)}{dt} = B - \Lambda S(t) - \mu_S S(t)$$

Generic SIR model

$$\frac{dS(t)}{dt} = B - \Lambda S(t) - \mu_S S(t)$$

$\Lambda = \Lambda(S, I, R)$: **Force of infection**

Generic SIR model

$$\frac{dS(t)}{dt} = B - \Lambda S(t) - \mu_S S(t)$$

$$\frac{dI(t)}{dt} = \Lambda S(t) - qI(t) - \mu_I I(t)$$

Generic SIR model

$$\frac{dS(t)}{dt} = B - \Lambda S(t) - \mu_S S(t)$$

$$\frac{dI(t)}{dt} = \Lambda S(t) - qI(t) - \mu_I I(t)$$

$$\frac{dR(t)}{dt} = qI(t) - \mu_R R(t)$$

Generic SIR model

$$\frac{dS(t)}{dt} = -\Lambda S(t)$$

$$\frac{dI(t)}{dt} = \Lambda S(t) - qI(t)$$

$$\frac{dR(t)}{dt} = qI(t)$$

Generic SIR model

$$\frac{dS(t)}{dt} = -\Lambda S(t)$$

$$\frac{dI(t)}{dt} = \Lambda S(t) - qI(t)$$

$$\frac{dR(t)}{dt} = qI(t)$$

$$\frac{d}{dt}\{S(t) + I(t) + R(t)\} = 0$$

Generic SIR model

$$\frac{dS(t)}{dt} = -\Lambda S(t)$$

$$\frac{dI(t)}{dt} = \Lambda S(t) - qI(t)$$

$$\frac{dR(t)}{dt} = qI(t)$$

$$S(t) + I(t) + R(t) = N$$

(time-independent constant total population size)

Kermack-McKendrick SIR model

Kermack-McKendrick SIR model

$$\Lambda \propto I$$

Kermack-McKendrick SIR model

$$\Lambda \propto I$$

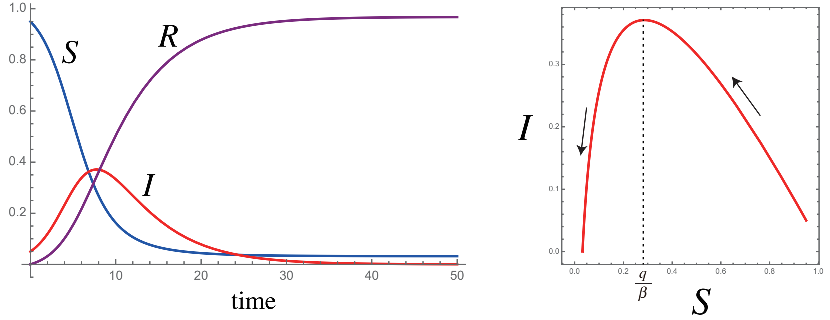
$$\frac{dS(t)}{dt} = -\beta I(t) S(t)$$

$$\frac{dI(t)}{dt} = \beta I(t) S(t) - qI(t)$$

$$\frac{dR(t)}{dt} = qI(t)$$

Epidemic dynamics model

Kermack-McKendrick SIR model



Kermack-McKendrick SIR model

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Kermack-McKendrick SIR model

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$$\frac{dI(t)}{dt} = \beta I(t)S(t) - qI(t)$$

$$\frac{dS(t)}{dt} + \frac{dI(t)}{dt} - \frac{q}{\beta}\frac{1}{S(t)}\frac{dS(t)}{dt} = 0$$

Kermack-McKendrick SIR model

$$\frac{dS(t)}{dt} = -\beta I(t)S(t)$$

$$\frac{dI(t)}{dt} = \beta I(t)S(t) - qI(t)$$

$$\frac{d}{dt}\left\{S(t) + I(t) - \frac{q}{\beta}\log S(t)\right\} = 0$$

Kermack-McKendrick SIR model

$$\frac{dS(t)}{dt} = -\beta I(t)S(t)$$

$$\frac{dI(t)}{dt} = \beta I(t)S(t) - qI(t)$$

$$S(t) + I(t) - \frac{q}{\beta}\log S(t) = \textbf{const.}$$

Kermack-McKendrick SIR model

$$\frac{dS(t)}{dt} = -\beta I(t)S(t)$$

$$\frac{dI(t)}{dt} = \beta I(t)S(t) - qI(t)$$

Conserved quantity of Kermack–McKendrick SIR model

$$S(t) + I(t) - \frac{q}{\beta}\log S(t) = S(0) + I(0) - \frac{q}{\beta}\log S(0)$$

Kermack-McKendrick SIR model

Conserved quantity of Kermack–McKendrick SIR model

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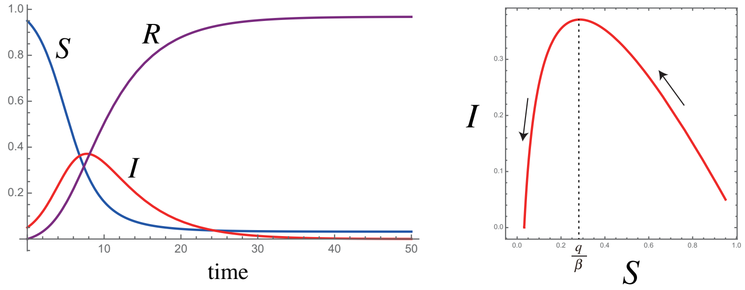
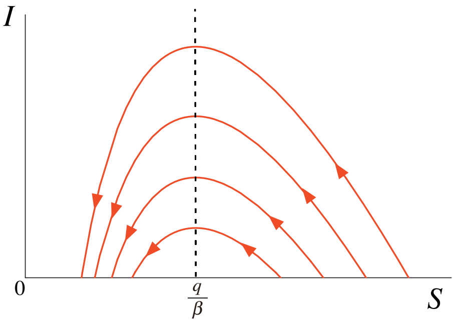


Kermack-McKendrick SIR model

Conserved quantity of Kermack–McKendrick SIR model

$$S(t) + I(t) - \frac{q}{\beta} \log S(t) = S(0) + I(0) - \frac{q}{\beta} \log S(0)$$



Kermack-McKendrick SIR model

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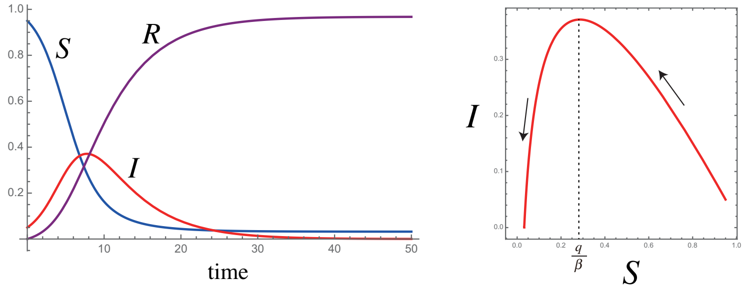
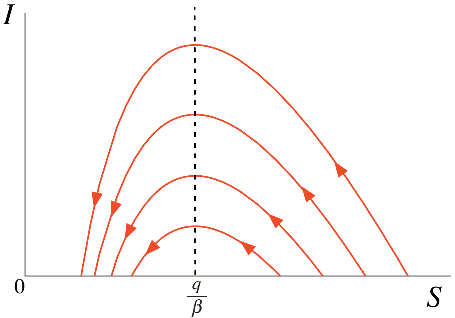


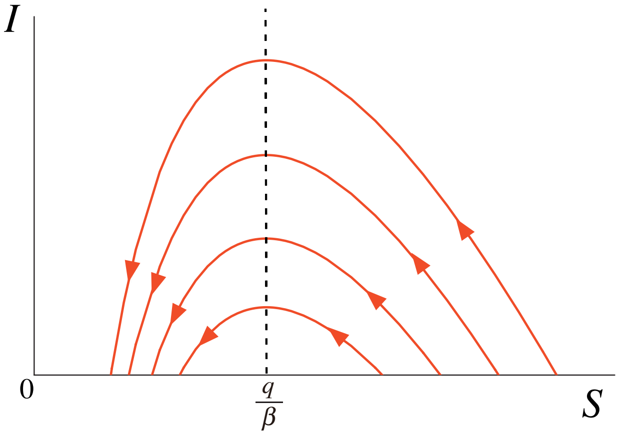
Kermack-McKendrick SIR model



Final size equation for Kermack–McKendrick SIR model

$$S_\infty - \frac{q}{\beta}\log S_\infty = S(0) + I(0) - \frac{q}{\beta}\log S(0)$$

Kermack-McKendrick SIR model



Kermack-McKendrick SIR model

$S(0) > \frac{q}{\beta} \quad \Rightarrow \quad I(t)$ increases in the early period of disease invasion;

$S(0) \leq \frac{q}{\beta} \quad \Rightarrow \quad I(t)$ monotonically decreases.

Kermack-McKendrick SIR model

$$\frac{\beta}{q} S(0) > 1 \quad \Rightarrow \quad$$ $I(t)$ increases in the early period of disease invasion;

$$\frac{\beta}{q} S(0) \leq 1 \quad \Rightarrow \quad$$ $I(t)$ monotonically decreases.

Kermack-McKendrick SIR model

$$\frac{\beta}{q} S(0) > 1 \quad \Rightarrow \quad I(t) \text{ increases in the early period of disease invasion;}$$

$$\frac{\beta}{q} S(0) \leq 1 \quad \Rightarrow \quad I(t) \text{ monotonically decreases.}$$

Basic reproduction number for Kermack–McKendrick SIR model

$$\mathscr{R}_0 := \frac{\beta}{q} S(0)$$

Kermack-McKendrick SIR model

$\mathscr{R}_0 > 1 \quad \Rightarrow \quad I(t)$ **increases in the early period of disease invasion;**

$\mathscr{R}_0 \leq 1 \quad \Rightarrow \quad I(t)$ **monotonically decreases.**

Basic reproduction number for Kermack–McKendrick SIR model

$$\mathscr{R}_0 := \frac{\beta}{q} S(0) \lessapprox \overline{\mathscr{R}}_0 := \frac{\beta}{q} N$$

Basic reproduction number $\mathscr{R}_0$

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In the biological context, the basic reproduction number $\mathscr{R}_0$ is defined as the expected number of new cases of an infection caused by an infective individual, in a population consisting of susceptible contacts only.

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※ The definition is independent of the epidemic situation in the population!

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※ The definition is independent of the epidemic situation in the population!

$\mathscr{R}_0 > 1 \quad \Rightarrow$ The number of infectives (likely) increases;

$\mathscr{R}_0 < 1 \quad \Rightarrow$ The number of infectives decreases.

Basic reproduction number $\mathscr{R}_0$

In the biological context, the basic reproduction number $\mathscr{R}_0$ is defined as the expected number of new cases of an infection caused by an infective individual, in a population consisting of susceptible contacts only.

※ The definition is independent of the epidemic situation in the population!

The number of infectives increases. $\Rightarrow$ $\mathscr{R}_0 > 1$;

The number of infectives decreases. $\Rightarrow$ $\mathscr{R}_0 < 1$ (likely)

Basic reproduction number $\mathscr{R}_0$

Kermack-McKendrick SIR model

$$\frac{dS(t)}{dt} = -\beta I(t)S(t)$$

$$\frac{dI(t)}{dt} = \beta I(t)S(t) - qI(t)$$

Basic reproduction number $\mathscr{R}_0$

Kermack-McKendrick SIR model

$$\frac{dI(t)}{dt} = \beta I(t)S(t) - qI(t)$$

Basic reproduction number $\mathscr{R}_0$

Kermack-McKendrick SIR model

$$\frac{dI(t)}{dt} = q\left\{\frac{\beta}{q}S(t) - 1\right\}I(t)$$

Basic reproduction number $\mathscr{R}_0$

Kermack-McKendrick SIR model

$$\frac{dI(t)}{dt} = q\left\{\frac{\beta}{q}S(t) - 1\right\}I(t)$$

$$\frac{\beta}{q}S(t) > 1 \quad \Rightarrow \quad I(t) \text{ increases;}$$

$$\frac{\beta}{q}S(t) < 1 \quad \Rightarrow \quad I(t) \text{ decreases.}$$

Basic reproduction number $\mathscr{R}_0$

Kermack-McKendrick SIR model

$$\frac{dI(t)}{dt} = q\left\{\frac{\beta}{q}S(t) - 1\right\}I(t)$$

Effective reproduction number $\mathscr{R}_t$ for Kermack–McKendrick SIR model

$$\mathscr{R}_t := \frac{\beta}{q}S(t)$$

Basic reproduction number $\mathscr{R}_0$

Kermack-McKendrick SIR model

$$\frac{dI(t)}{dt} = q\left\{\frac{\beta}{q}S(t) - 1\right\}I(t)$$

Effective reproduction number $\mathscr{R}_t$ for Kermack–McKendrick SIR model

$$\mathscr{R}_t := \frac{\beta}{q}S(t) \leq \overline{\mathscr{R}}_0 := \frac{\beta}{q}N$$

Epidemic dynamics model

William Ogilvy Kermack
(26 April 1898 – 20 July 1970)

Anderson Gray McKendrick
(8 September 1876 – 30 May 1943)

References: Davidson, J. N. (1971) "William Ogilvy Kermack. 1898-1970". Biographical Memoirs of Fellows of the Royal Society, 17: 399–429. doi:10.1098/rsbm.1971.0015; http://www-history.mcs.st-and.ac.uk/Biographies/McKendrick.html

Epidemic dynamics model

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- Kermack, W, McKendrick, A (1991) Contributions to the mathematical theory of epidemics – II. The problem of endemicity. Bulletin of Mathematical Biology, 53(1-2): 57–87. doi:10.1007/BF02464424.
  Reprinted from
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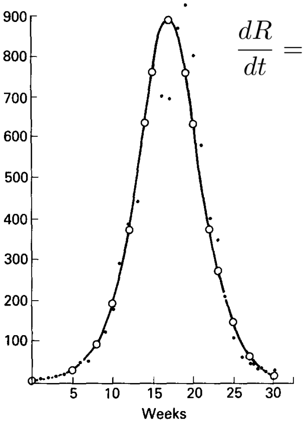
- Kermack, W, McKendrick, A (1991) Contributions to the mathematical theory of epidemics – III. Further studies of the problem of endemicity. Bulletin of Mathematical Biology, 53(1-2): 89–118. doi:10.1007/BF02464425.
  Reprinted from
  Proceedings of the Royal Society, Vol. 141A, pp. 94–122 (1933)

Epidemic dynamics model

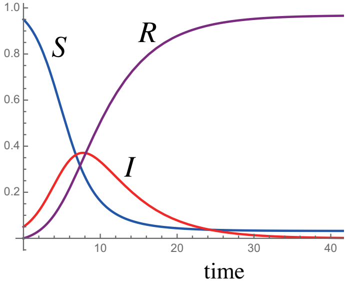


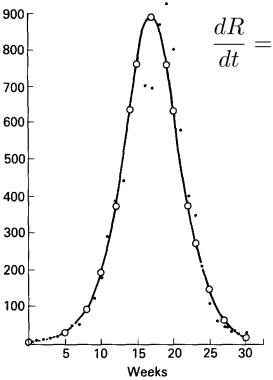
$$\frac{dR}{dt} = qI$$

Figure 1. Deaths from plague in the island of Bombay over the period 17 December 1905 to 21 July 1906. The ordinate represents the number of deaths per week, and the abscissa denotes the time in weeks. As at least 80–90% of the cases reported terminate fatally, the ordinate may be taken as approximately representing $\mathrm{d}z/\mathrm{d}t$ as a function of t. The calculated curve is drawn from the formula:

$$y = \frac{\mathrm{d}z}{\mathrm{d}t} = 890 \operatorname{sech}^2(0.2t - 3.4).$$

Epidemic dynamics model





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Epidemic dynamics model

- H. Inaba, Age-Structured Population Dynamics in Demography and Epidemiology, Springer, Singapore, 2017.
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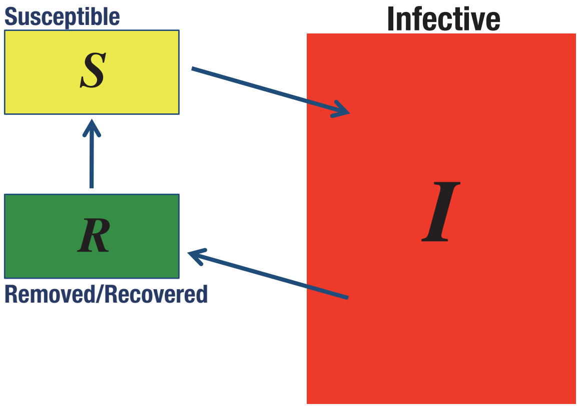
Kermack-McKendrick SIRS model

Kermack-McKendrick SIRS model



Kermack-McKendrick SIRS model

$$\frac{dS(t)}{dt} = -\beta I(t)S(t) + \omega R(t)$$

$$\frac{dI(t)}{dt} = \beta I(t)S(t) - qI(t)$$

$$\frac{dR(t)}{dt} = qI(t) - \omega R(t)$$

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$$S(t) + I(t) + R(t) = N$$

Epidemic dynamics model

Kermack-McKendrick SIRS model

$$\overline{\mathscr{R}}_0 := \frac{\beta}{q} N$$

$$\frac{dS(t)}{dt} = -\beta I(t) S(t) + \omega R(t)$$

$$\frac{dI(t)}{dt} = \beta I(t) S(t) - q I(t)$$

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$$S(t) + I(t) + R(t) = N$$

Kermack-McKendrick SIRS model

$$\overline{\mathscr{R}}_0 := \frac{\beta}{q} N$$

$$\frac{dS(t)}{dt} = -\beta I(t)S(t) + \omega\{N - S(t) - I(t)\}$$

$$\frac{dI(t)}{dt} = \beta I(t)S(t) - qI(t)$$

Kermack-McKendrick SIRS model

$$\overline{\mathscr{R}}_0 := \frac{\beta}{q} N$$

$$\overline{\mathscr{R}}_0 \leq 1 \;\Rightarrow (S(t), I(t)) \underset{t\to\infty}{\longrightarrow} (N, 0)$$

Kermack-McKendrick SIRS model

$$\overline{\mathscr{R}}_0 := \frac{\beta}{q} N$$

$$\overline{\mathscr{R}}_0 \leq 1 \quad \Rightarrow (S(t), I(t)) \underset{t \to \infty}{\longrightarrow} (N, 0)$$

$$\overline{\mathscr{R}}_0 > 1 \quad \Rightarrow (S(t), I(t)) \underset{t \to \infty}{\longrightarrow} (\frac{q}{\beta}, I^*)$$

$$I^* = \frac{1 - 1/\overline{\mathscr{R}}_0}{1 + q/\omega} N$$

Kermack-McKendrick SIRS model

$$\overline{\mathscr{R}_0} := \frac{\beta}{q} N$$

$$\overline{\mathscr{R}_0} \leq 1 \quad \Rightarrow (S(t), I(t)) \underset{t\to\infty}{\longrightarrow} (N, 0)$$

【感染症不在平衡状態】
disease-free equilibrium state

$$\overline{\mathscr{R}_0} > 1 \quad \Rightarrow (S(t), I(t)) \underset{t\to\infty}{\longrightarrow} (\frac{q}{\beta}, I^*)$$

$$I^* = \frac{1 - 1/\overline{\mathscr{R}_0}}{1 + q/\omega} N$$

Kermack-McKendrick SIRS model

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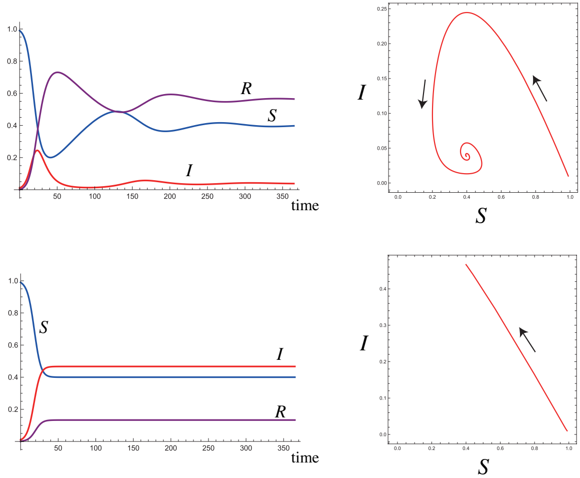
【感染症不在平衡状態】
disease-free equilibrium state

$$\overline{\mathscr{R}_0} > 1 \quad \Rightarrow (S(t), I(t)) \underset{t\to\infty}{\longrightarrow} (\frac{q}{\beta}, I^*)$$

【感染症定着平衡状態】
endemic equilibrium state

$$I^* = \frac{1 - 1/\overline{\mathscr{R}_0}}{1 + q/\omega} N$$

Kermack-McKendrick SIRS model



$\overline{\mathscr{R}}_0 = 2.5$;

$q = \frac{1}{7}$;

$\omega = 0.1,\ 0.5$;

$$(S(0), I(0), R(0)) = (0.99N, 0.01N, 0.0).$$

Force of infection

Force of infection

Kermack–McKendric model

$$\Lambda = \beta I$$

Force of infection

Kermack–McKendric model

$$\Lambda = \beta I = \gamma \cdot \frac{I}{\widetilde{N}} \cdot c\widetilde{N}$$

Force of infection

Kermack–McKendric model

$$\Lambda = \beta I = \gamma \cdot \frac{I}{\widetilde{N}} \cdot c\widetilde{N}$$

Frequency/Ratio-dependent force of infection

$$\Lambda = \lambda \frac{I}{\widetilde{N}}$$

Force of infection

Kermack–McKendric model

$$\Lambda = \beta I = \gamma \cdot \frac{I}{\widetilde{N}} \cdot c\widetilde{N}$$

Frequency/Ratio-dependent force of infection

$$\Lambda = \lambda \frac{I}{\widetilde{N}} = \gamma \cdot \frac{I}{\widetilde{N}} \cdot C$$

Force of infection

Vector-borne disease transmission (mass-action type)

$$\Lambda = \widehat{\beta}_{\mathrm{H}} V$$

Force of infection

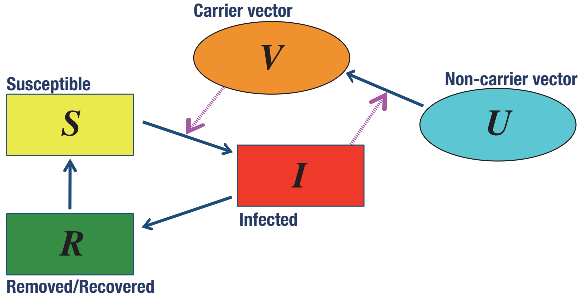
Vector-borne disease transmission (mass-action type)

$$\Lambda = \widehat{\beta}_{\mathrm{H}} V = \widehat{\gamma} \cdot \frac{V}{\widetilde{M}} \cdot \widehat{c}\widetilde{M}$$

Force of infection

Vector-borne disease transmission (mass-action type)

$$\Lambda = \widehat{\beta}_{\mathrm{H}} V$$



Epidemic dynamics model

A model for the vector-borne disease transmission

$$
\begin{aligned}
\frac{dS(t)}{dt} &= -\widehat{\beta}_{\mathrm{H}} V(t) S(t) + \omega R(t) \\
\frac{dI(t)}{dt} &= \widehat{\beta}_{\mathrm{H}} V(t) S(t) - q I(t) \\
\frac{dR(t)}{dt} &= q I(t) - \omega R(t) \\
\frac{dU(t)}{dt} &= Q - \widehat{\beta}_{\mathrm{M}} I(t) U(t) - \delta U(t) \\
\frac{dV(t)}{dt} &= \widehat{\beta}_{\mathrm{M}} I(t) U(t) - \delta V(t)
\end{aligned}
$$

Epidemic dynamics model

A model for the vector-borne disease transmission

$$\frac{dS(t)}{dt} = -\widehat{\beta}_{\mathrm{H}} V(t) S(t) + \omega \{ N - S(t) - I(t) \}$$

$$\frac{dI(t)}{dt} = \widehat{\beta}_{\mathrm{H}} V(t) S(t) - q I(t)$$

$$\frac{dV(t)}{dt} = \widehat{\beta}_{\mathrm{M}} I(t) \left\{ \frac{Q}{\delta} - V(t) \right\} - \delta V(t)$$

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Basic reproduction number

$$\overline{\mathscr{R}}_0 := \underbrace{\left(\widehat{\beta}_{\mathrm{H}} N \cdot \frac{1}{\delta} \right)}_{\text{human infection by a carrier vector}} \times \underbrace{\left(\widehat{\beta}_{\mathrm{M}} \frac{Q}{\delta} \cdot \frac{1}{q} \right)}_{\text{production of carrier vectors by an infective}}$$

Epilogue:

Various factors on the epidemic dynamics

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- **Route of disease transmission;**

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Various factors on the epidemic dynamics

- Route of disease transmission;
- Condition of public health;
- Condition of medical treatment;
- Cultural/social custom in the daily life;

Various factors on the epidemic dynamics

- Route of disease transmission;
- Condition of public health;
- Condition of medical treatment;
- Cultural/social custom in the daily life;
- Social response under the cultural/political/economic background.

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