

# On Travelling Wave of Disease Infection with Diffusing Epidemic Vector

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In our mathematical model, we apply the idea of Kermack-McKendrick model for the dynamical relation between host and epidemic vector, and to introduce the spatial propagation of epidemic disease, we assume the random diffusion of host and vector in infinite one-dimensional space:

$$\left\{ \begin{array}{l} \frac{\partial v_+(x,t)}{\partial t} = D_v \frac{\partial^2 v_+(x,t)}{\partial x^2} + \alpha h_+(x,t)v_-(x,t) \\ \frac{\partial v_-(x,t)}{\partial t} = D_v \frac{\partial^2 v_-(x,t)}{\partial x^2} - \alpha h_+(x,t)v_-(x,t) \\ \frac{\partial h_+(x,t)}{\partial t} = D_h \frac{\partial^2 h_+(x,t)}{\partial x^2} + \beta h_-(x,t)v_+(x,t) - \gamma h_+(x,t) \\ \frac{\partial h_-(x,t)}{\partial t} = D_h \frac{\partial^2 h_-(x,t)}{\partial x^2} - \beta h_-(x,t)v_+(x,t) + \gamma(1 - \epsilon)h_+(x,t). \end{array} \right.$$

Susceptible host  $h_-$  is infected by contact to infectious vectors  $v_+$  with transmission rate  $\alpha$ , while the susceptible vector  $v_-$  is done by contact to infectious hosts  $h_+$  with transmission rate  $\beta$ . Parameter  $\gamma$  is the recovery rate for infectious host. Recovered host can get the immunity with probability  $\epsilon$ , so that host without the immunity returns to susceptible with probability  $1 - \epsilon$ .  $D_v$  is the diffusion coefficient for vector population, and  $D_h$  that for host one. We assume that the disease infection does not affect the mobility of host and vector, so that infectious and susceptible have the same diffusion coefficient, although vector and host have different in general.

In our model, we focus the existence and the propagating speed of stationary travelling epidemic wave, and analyze the above-shown model by means of mathematically analytical and numerical methods. We try to consider the biological meanings of those obtained mathematical results and discuss the characteristics of epidemic travelling wave.