A POPULATION DYNAMICS MODEL FOR INFORMATION SPREAD UNDER THE EFFECT OF SOCIAL RESPONSE

Emmanuel Jesuyon Dansu*1 and Hiromi Seno2

1Department of Mathematical Sciences, Federal University of Technology, Akure, Nigeria
2Research Center for Pure and Applied Mathematics, Graduate School of Information Sciences, Tohoku University, Sendai, Japan
ejdansu@futa.edu.ng (*corresponding author), seno@math.is.tohoku.ac.jp

Mark Granovetter [1, 2, 3] promoted the threshold model of social behavior in which the acceptance value of one of two distinct actions is determined by the proportion of a given population that has already accepted the action. It is about the thinking that an individual embraces an idea once a sufficient number of people has embraced the idea. This model finds application in biology, sociology, economics, information science and lots more. In this study, we develop a population dynamics model based on Granovetter’s threshold hypothesis. We consider the possibility of an individual accepting and spreading some information given that a satisfactory proportion of people (threshold population) in their community is already doing the same. Given the frequency/proportion $P(t)$ of knowers of the information at a given time and the strength of social recognition effect $Q = Q(P)$ of the information, we assume that each individual is characterized by a threshold value $\xi$ for $Q$, independent of time, such that

\[
\begin{cases}
\xi \leq Q & \text{The individual may accept the information to transmit to others;} \\
\xi > Q & \text{The individual ignores the information.}
\end{cases}
\]

The differential equation model representing this information spread behavior is given as

\[
\frac{dP}{dt} = B(P(t)) \left[ 1 - P(t) - \int_{\Xi(P(t))} \{1 - \theta(\xi)\} f(\xi) d\xi \right],
\]

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where $B(P)$ is the coefficient of information transmission to non-knowers who are willing to accept the information; $\Xi(P)$ is the set of threshold values for which people are not yet willing to accept the information; $\theta(\xi)$ determines the ratio of initial knowers in the subpopulation with the threshold value $\xi$, such that $0 \leq \theta(\xi) \leq 1$; $f(\xi)$ is the frequency distribution function (FDF) for the threshold value $\xi$ in the population. Our analyses show that the final proportion of knowers of the information is determined by the initial proportion of knowers. We also see the existence of critical values for the initial knower size, the mean threshold value and the variance of threshold values. These critical values tend to have drastic impact on the proportion of the population that end up knowing the information.

References


A Population Dynamics Model for Information Spread Under the Effect of Social Response

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*Department of Mathematical Sciences, Federal University of Technology, Akure, Nigeria
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Background

Mark Granovetter promoted the concept of threshold model of social behavior in which the acceptance value of one of two distinct actions is determined by the proportion of a given population that have already accepted the action. It is about the thinking that an individual embraces an idea once a sufficient number of people have embraced it. This model finds application in sociology, economics, information science and lots more. In this study, we develop a population dynamics model based on Granovetter’s threshold hypothesis. In our study, we consider the possibility of an individual accepting and spreading some information given that a satisfactory proportion of people (threshold population) in their community are already doing the same. Our analyses show that the final proportion of knowers of the information is determined by the initial proportion of knowers. We also see the existence of critical values for the initial knower size, the mean threshold value and the variance of threshold values. These critical values tend to have drastic impact on the proportion of the population that end up knowing the information. See [1, 2, 3].

The threshold model

The general threshold distribution model for the dynamics of information spread is given as

\[ \frac{dP(t)}{dt} = B(P(t)) \left( 1 - P(t) - \int \phi(t) f(t) dt \right), \]  

(1)

The model is formulated with the following variables and parameters.

- \( B \): the frequency of non-knowers in the population at time \( t \).
- \( \phi(t) \): the frequency distribution function (PDF) of knower’s threshold value \( \xi \) in the population.
- \( f(t) \): the cumulative distribution function (CDF) of the threshold value \( \xi \) in the population.
- \( \phi(t) : f(t) \): the cumulative distribution function (CDF) of the threshold value \( \xi \) in the population.

In this case, the distribution of \( \xi \) is uniform with \( f(t) \) given as

\[ f(t) = \begin{cases} 0, & \xi < 0, \\ \frac{1}{2}, & 0 \leq \xi \leq 2c, \\ 0, & \xi > 2c. \end{cases} \]  

(2)

with mean \( c \).

Model with compact support frequency distribution

In Fig. 1: Graph of the frequency distribution function \( f(t) \) against the threshold value \( \xi \) of the social recognition effect given by (2).

The model is expressed as

\[ \frac{dP(t)}{dt} = B(P(t)) \left( 1 - P(t) - \int \phi(t) f(t) dt \right), \]  

(3)

Eq. 3 numerically obtained convergence for \( P(t) \) with parameter \( c = 0.0, 0.2, 0.4, 0.6, 0.8, 1.0 \).

Fig. 4: Numerically obtained convergence for \( P(t) \) with parameter \( c = 0.0, 0.2, 0.4, 0.6, 0.8, 1.0 \).

The threshold model

where we now have

\[ f(t) = \begin{cases} 0, & \xi < \frac{P(t)}{1 - P(t)} - \frac{1}{2} (1 - \theta_0), \\ \frac{1}{2}, & \frac{P(t)}{1 - P(t)} - \frac{1}{2} (1 - \theta_0) \leq \xi, \\ 0, & \xi > \frac{P(t)}{1 - P(t)} - \frac{1}{2} (1 - \theta_0). \end{cases} \]  

(4)

with initial condition \( P(0) = P_0 \) and

\[ G(P) := 1 - P \left( 1 - \frac{1}{1 - \theta_0} \right) \int_{P(t)}^{\infty} \frac{f(t) dt}{dP(t)} \]  

(5)

Model with everywhere positive distribution

\( G(P) \) is continuous in \( P \) and \( f(t) \) is positive for every real threshold value \( \xi \). At the equilibrium state \( P = P^* \) where \( dP(t)/dt = 0 \), we have \( G(P^*) = 0 \) since \( B(P^*) > 0 \). With the specific distribution

\[ f(t) = \begin{cases} 0, & \xi < \xi, \\ \frac{1}{2}, & \xi \leq \xi, \\ 0, & \xi > \xi. \end{cases} \]  

(6)

Concluding remarks

The final proportion/frequency of knowers is largely dependent on the initial proportion of knowers. Critical conditions relating to heterogeneity of individuals in a population determine the consequence of information spread. For proper information dissemination, it is necessary to take into account the social situation of a population and the nature of information under consideration.

References