A Model for the Effect of Limited Isolation Capacity on the Final Epidemic Size

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Isolation/quarantine is one of the proper strategies for the reduction of the risk about the propagation of infectious diseases in the community. In this work, we consider a mathematical model with a system of ordinary differential equations to see how the limited isolation capacity affects the final epidemic size defined as the total number of infected individuals at the end of the season. The population is divided into four classes: those who are healthy and can be infected $S(t)$; those who are infected and are able to transmit the disease $I(t)$; those who are isolated and cannot contact with other individuals $Q(t)$; those who are immune after been infected and recovered $R(t)$. We assume that the total population size of the community is constant with no demographic change due to birth, death and migration. In this theoritical work, the isolation is kept within the time scale of epidemic dynamics.

$$
\begin{aligned}
\frac{dS}{dt} &= \frac{-\beta IS}{N-Q} \\
\frac{dI}{dt} &= \frac{\beta IS}{N-Q} - \gamma I - \sigma(Q)I \\
\frac{dQ}{dt} &= \sigma(Q)I \\
\frac{dR}{dt} &= \gamma I,
\end{aligned}
$$

where

$$
\sigma(Q) = \begin{cases} \sigma_0 & \text{for } Q < Q_{max} \\ 0 & \text{for } Q = Q_{max} \end{cases}
$$

The infection coefficient β is a positive constant, and the positive parameter γ denotes the natural recovery rate of the infective individual. The parameter Q_{max} means the capacity of isolation. The piecewise function $\sigma(Q)$ denotes the isolation rate of the infected individual, which means that, once the isolated subpopulation size reaches the limit Q_{max}, the isolation stops.

By analyzing our model to get the equation determining the final size, we try to discuss how the limited isolation capacity could affect the final epidemic size.